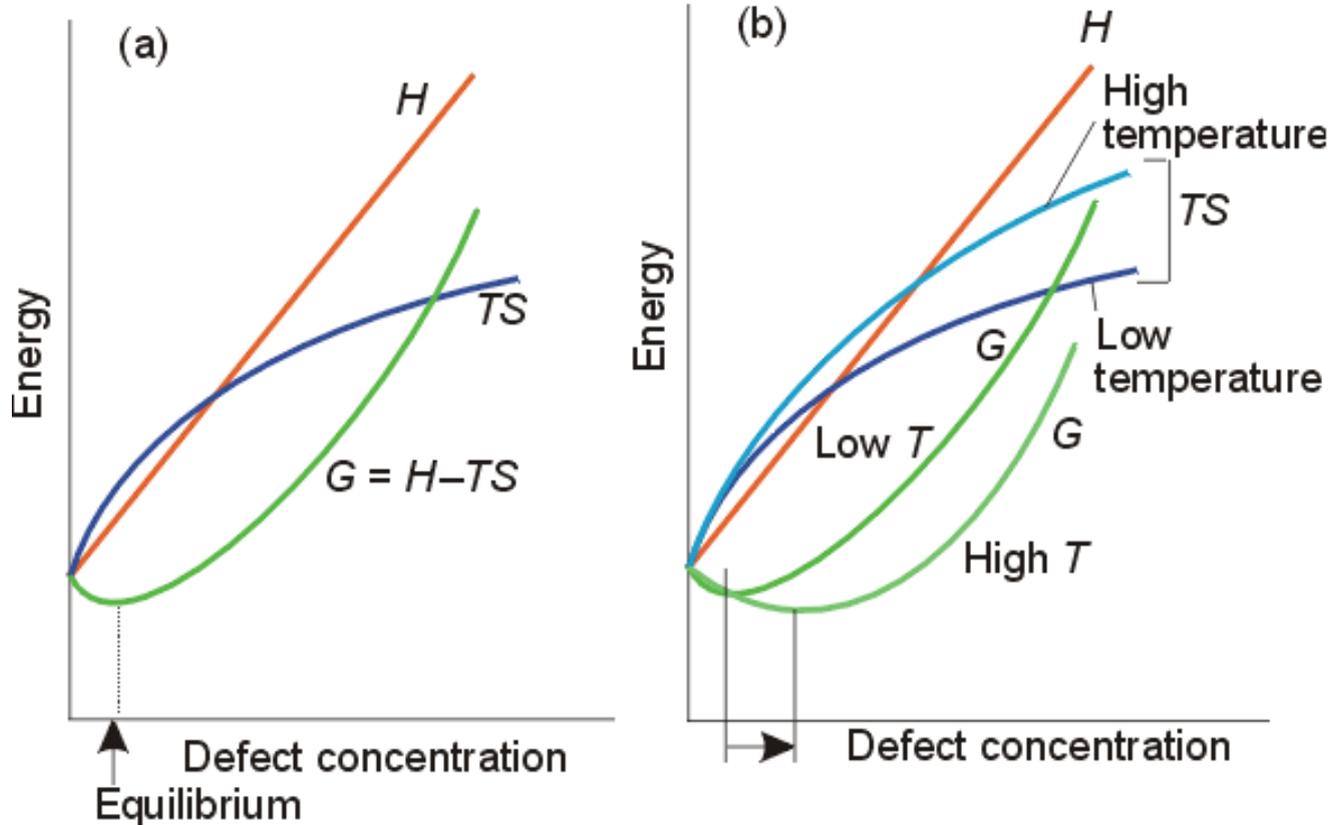


Chapter 18

Structures and Properties of Solids

Defects in solids



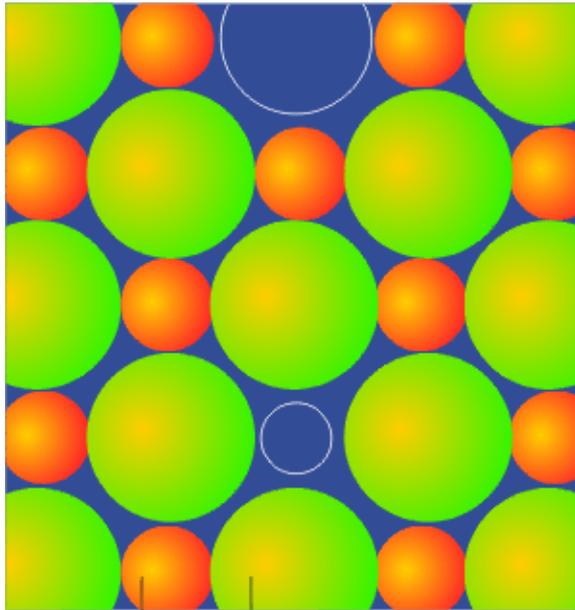
$$G = H - TS$$

Structural Defects

- Intrinsic defects**- occur in the pure substance
- Extrinsic defects**- stem from the presence of impurity

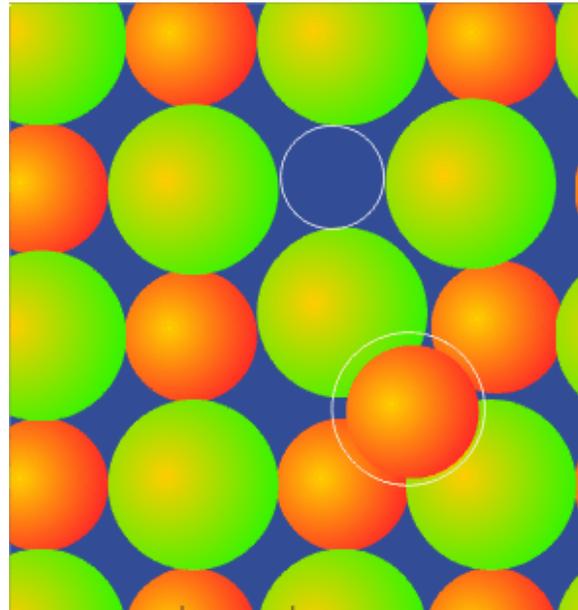
- Point defects**
- Extended defects**- in one, two or three dimension

Intrinsic Point Defects



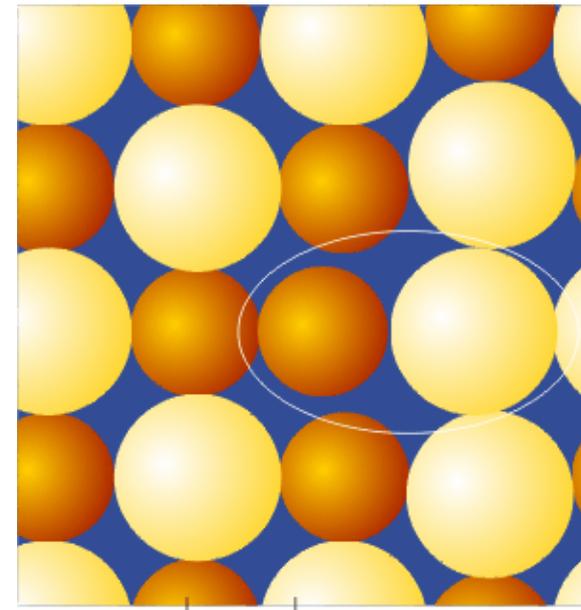
Shottky defect-

Missing ion-pair



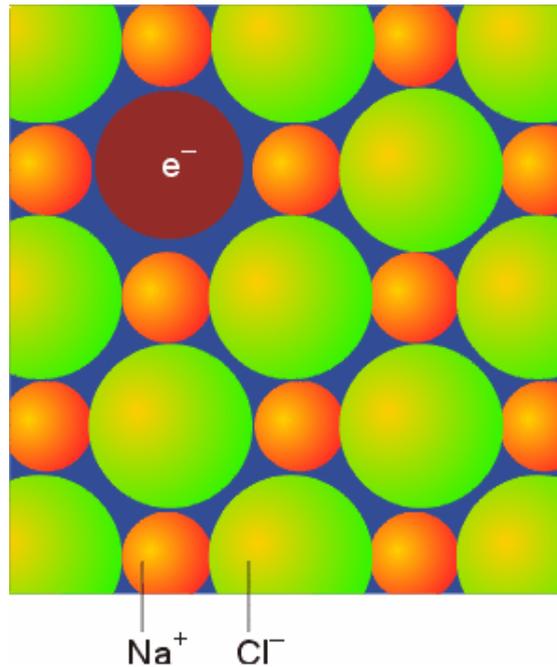
Frenkel defect-

misplacing ions to
interstitial



Atom exchange defect

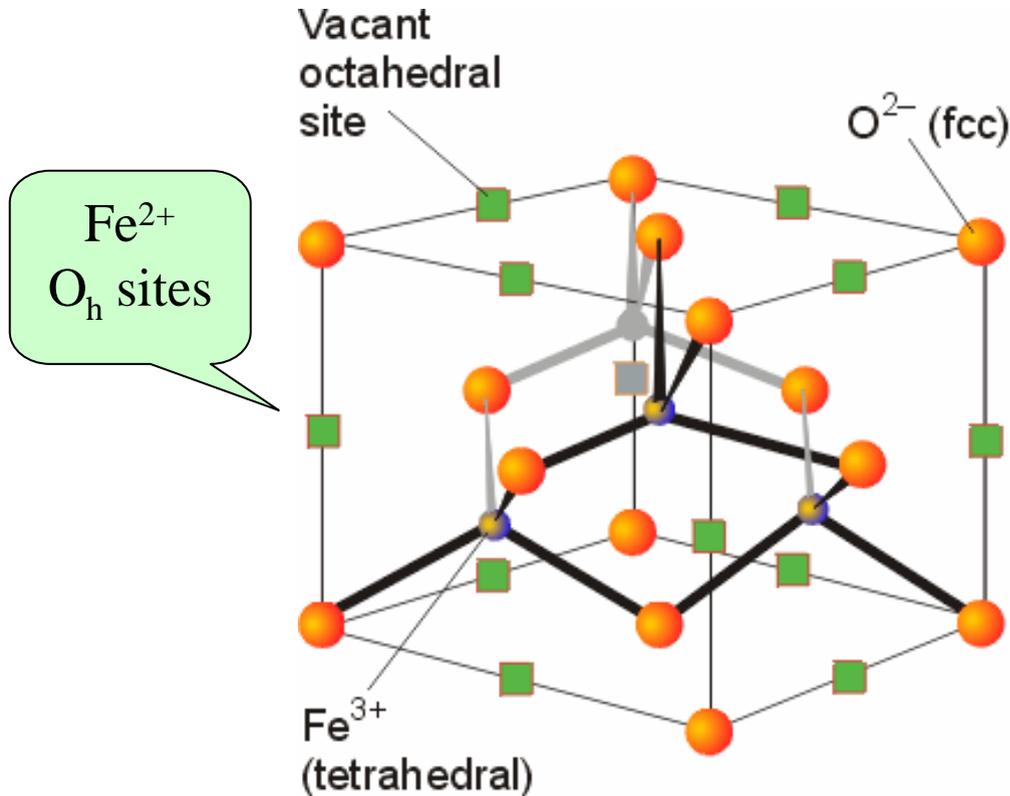
Extrinsic Point Defects



“F-center” defect

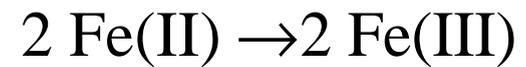
“*Farbenzenter*” German word for color center 18.6

Defect sites of FeO

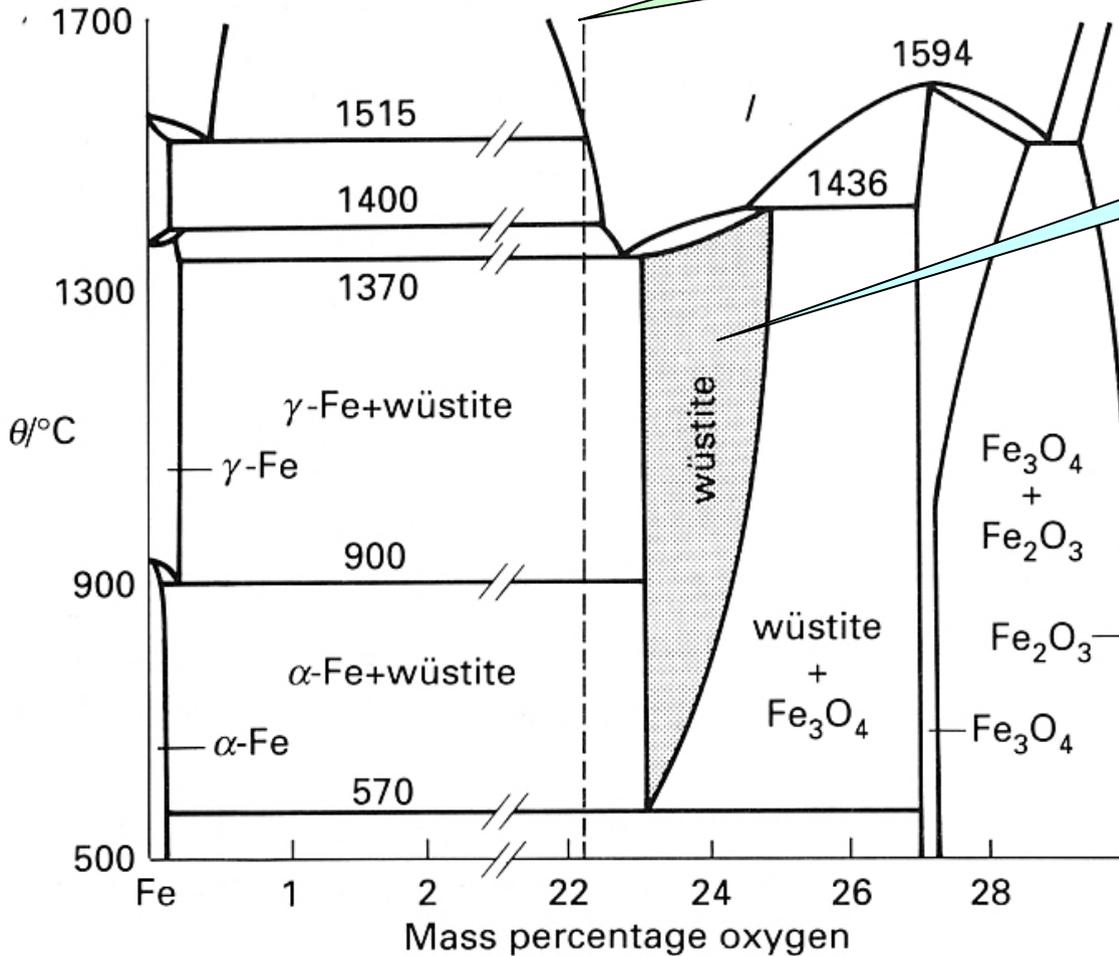


Nonstoichiometry of FeO_x

One Fe(II) vacancy is charge-compensated by



Stoichiometry FeO

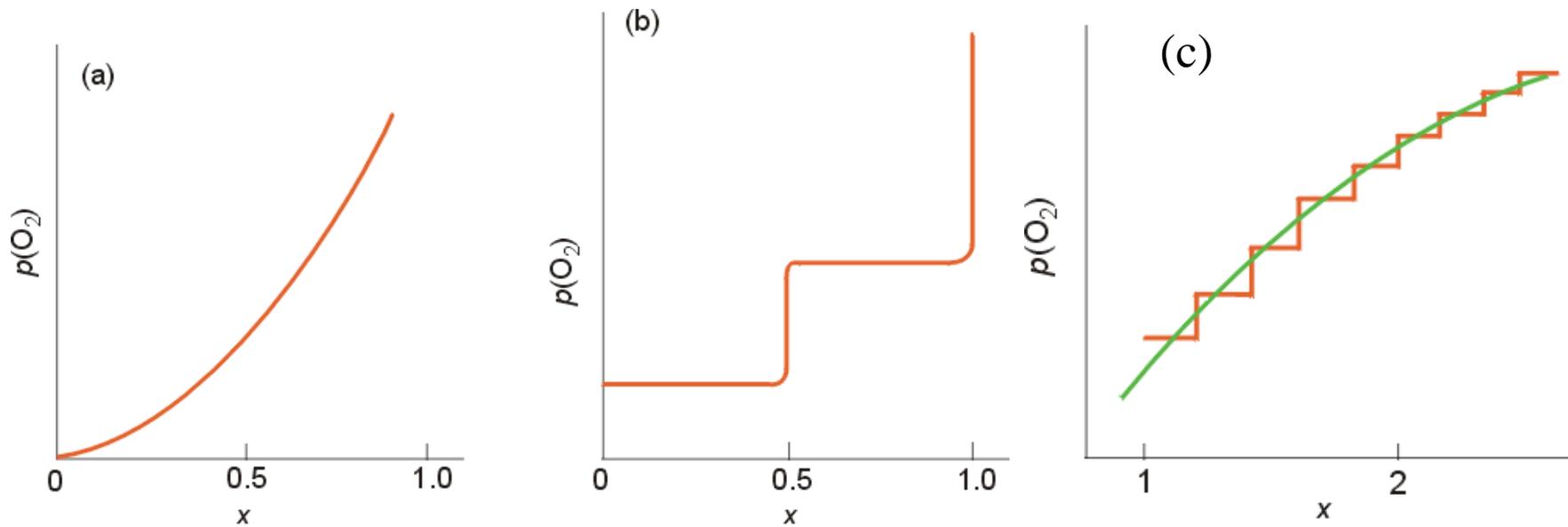


FeO_x



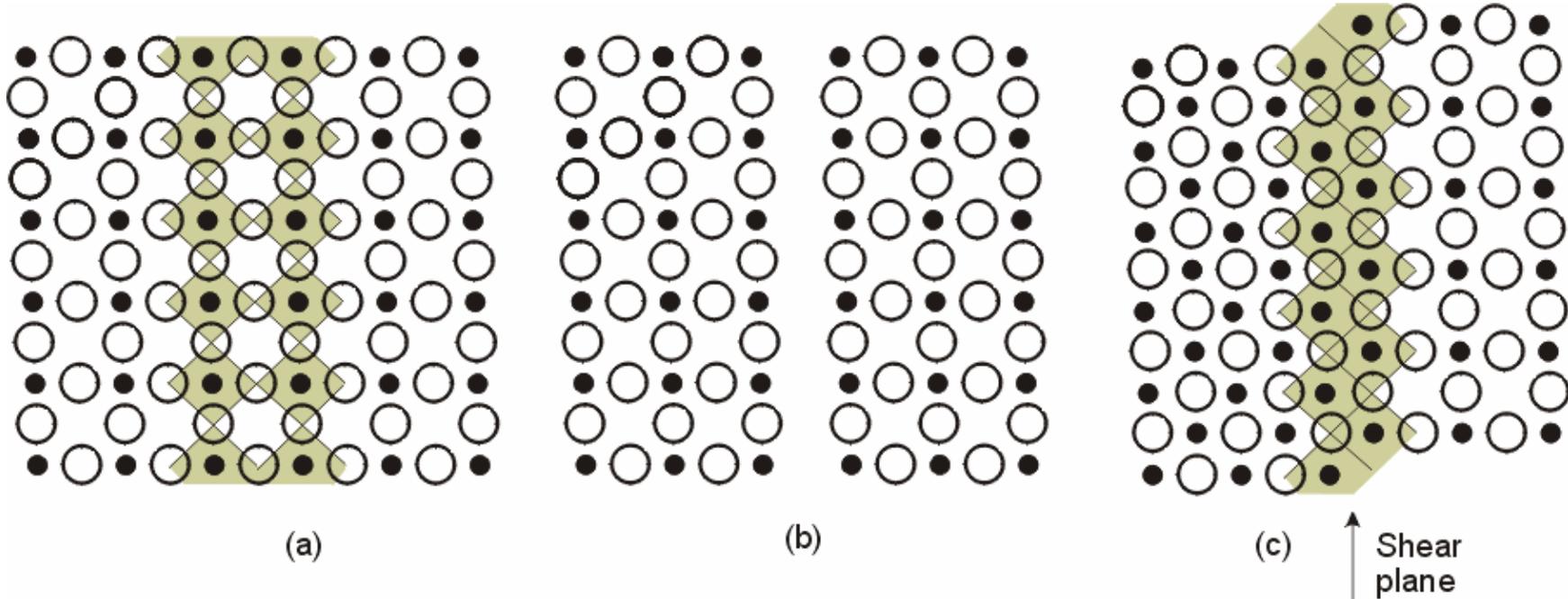
Jeff Weissman/ photographic Guide to Mineral

Phase diagram of Iron Oxides as a Function of Oxygen Content

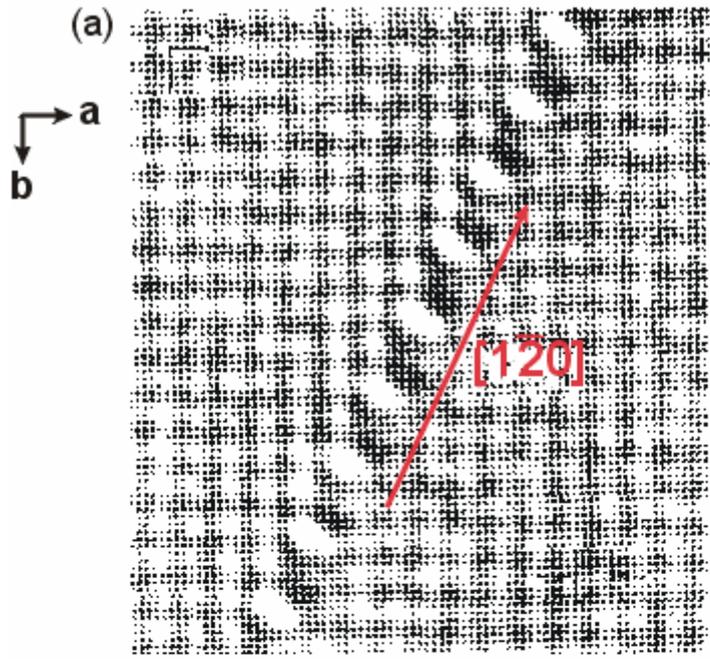


Variation of partial pressure of oxygen with composition at constant pressure for (a) a nonstoichiometric oxide; (b) a stoichiometric pair of metal oxides MO and MO_2 ; (c) a series of closely spaced discrete phases. The axis x is the atom ratio in MO_x .

Plane Defects

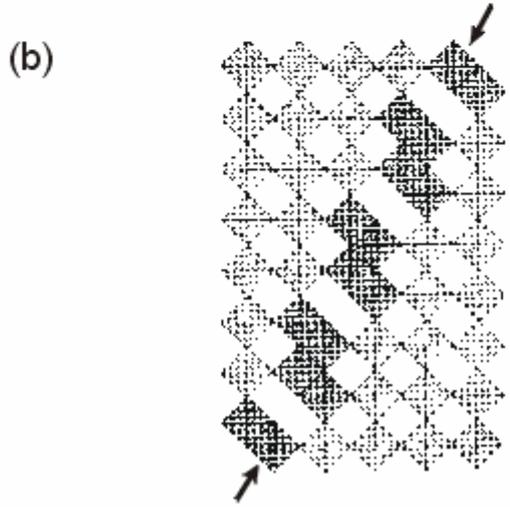


Shear plane: formation upon “Oxygen- removal”



Shear plane

**High resolution SEM
on $(\bar{1}20)$ in WO_{3-x}**



Insertion of oxygen

Group , , and transition metal oxides are good catalysts for selective oxidation of hydrocarbons.

e.g. V_2O_5 , MoO_3 , WO_3

Reasons for high activity and selectivity in the insertion of oxygen

Formation of a oxygen vacancy or a shear plane?

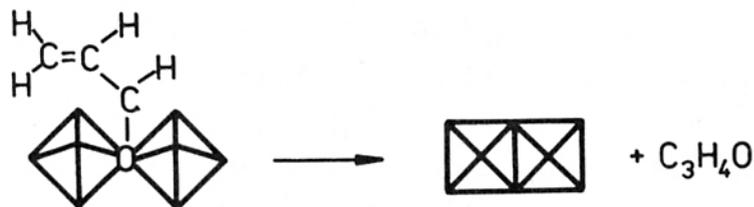
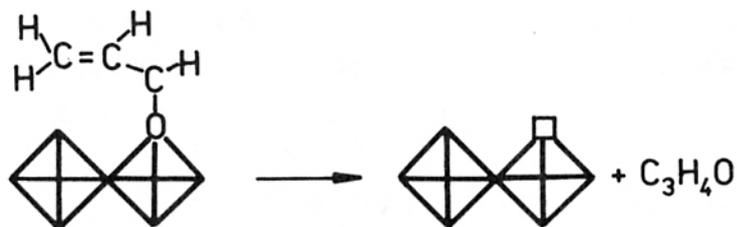
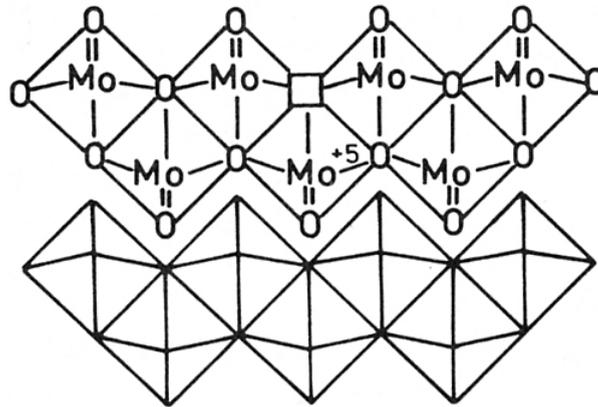
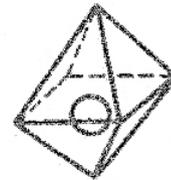
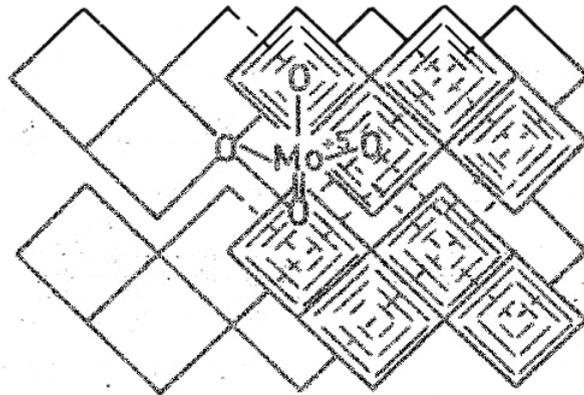


Figure 9. Mechanism of the insertion of oxygen into hydrocarbon molecule on oxide catalysts with point defects (a) and shear structures (b).

Surface structure



$$\begin{aligned}g_x &= 1,943 \\g_y &= 1,960 \\g_z &= 1,868\end{aligned}$$

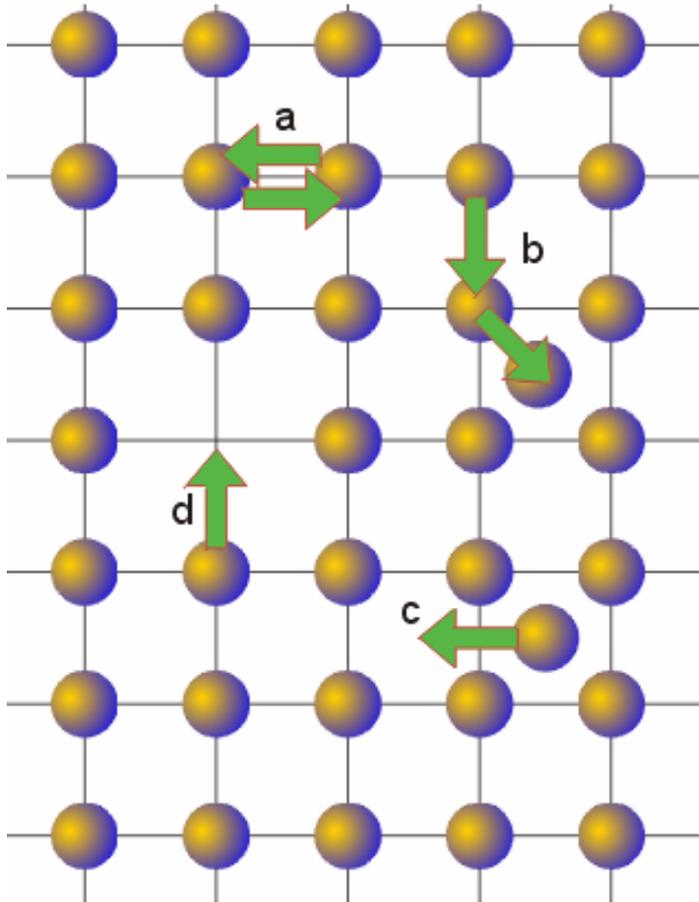


$$\begin{aligned}g_x &= 1,943 \\g_y &= 1,954 \\g_z &= 1,879\end{aligned}$$

Observed in
experiments

Figure 7. Generation of oxygen vacancies and shear planes in MoO₃ structure.

Ion Conductivity



**Diffuse mechanisms
for ions or atoms in a
solid:**

(a) exchange

(b) interstitialcy

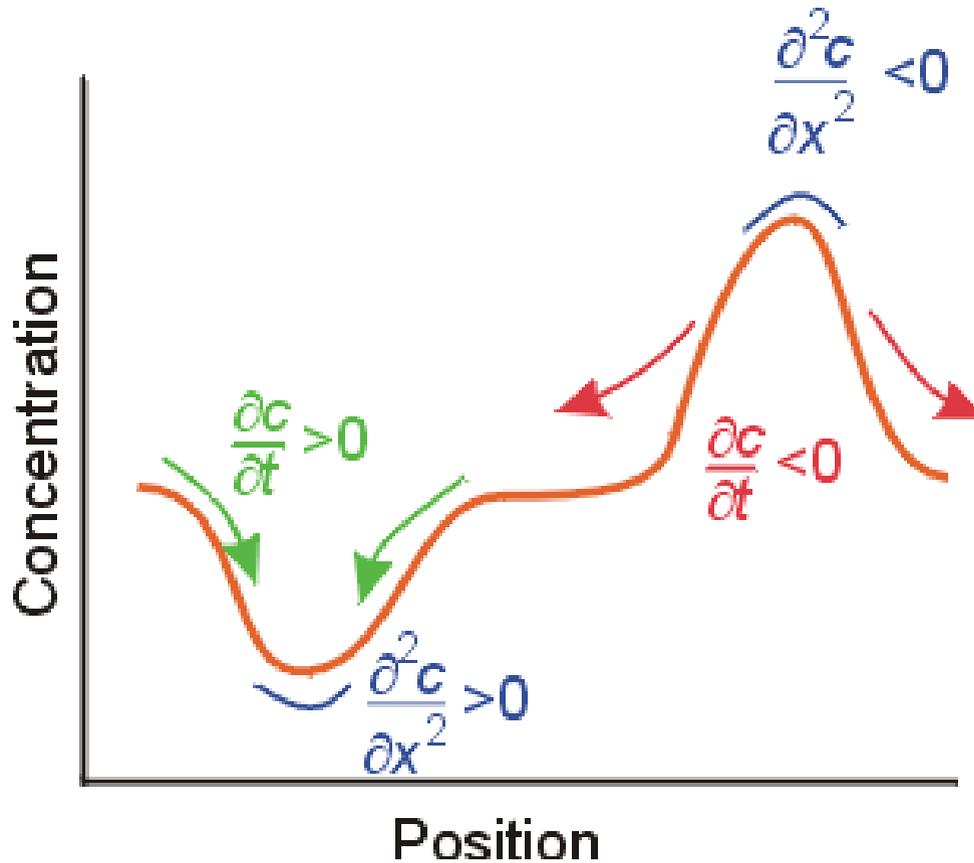
(c) interstitial

(d) vacancy

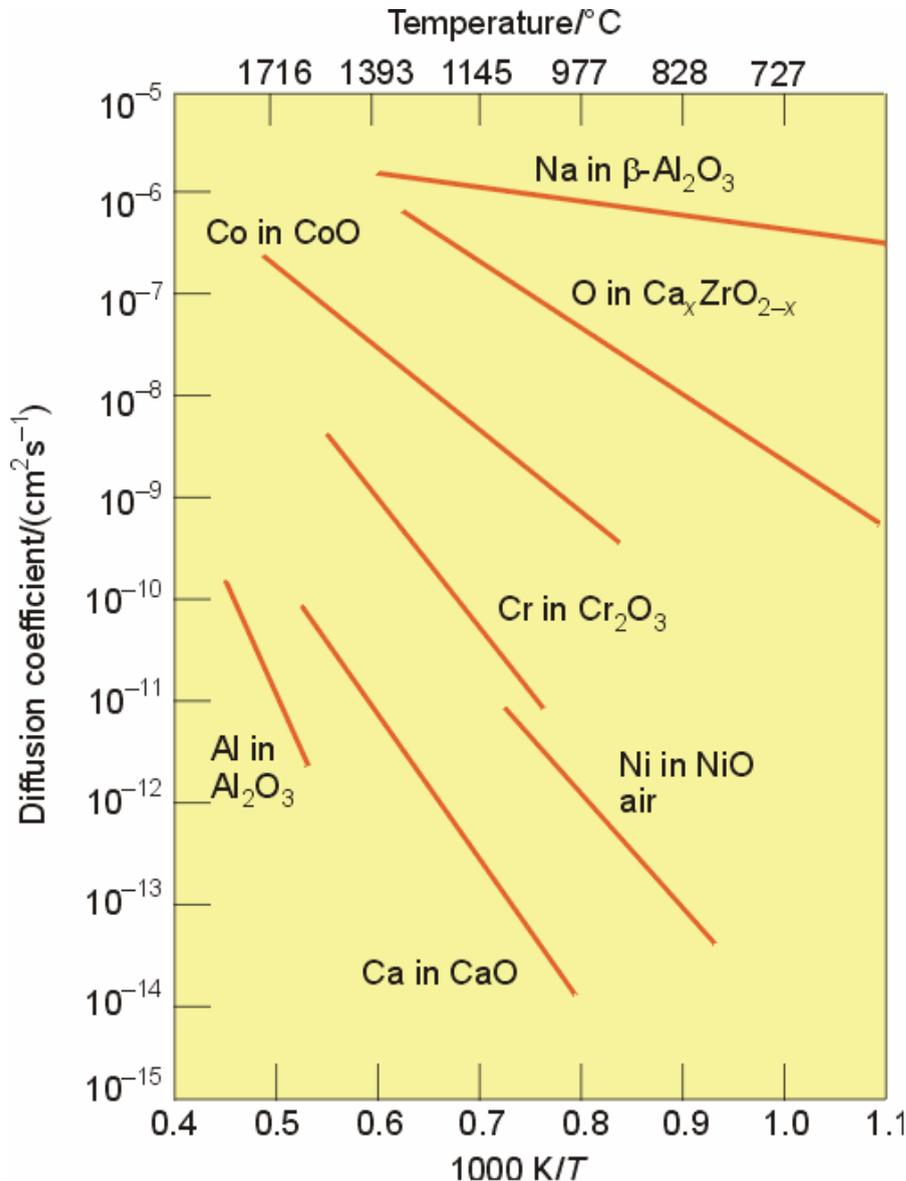
Diffusion

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2}$$

D : Diffusion coefficient

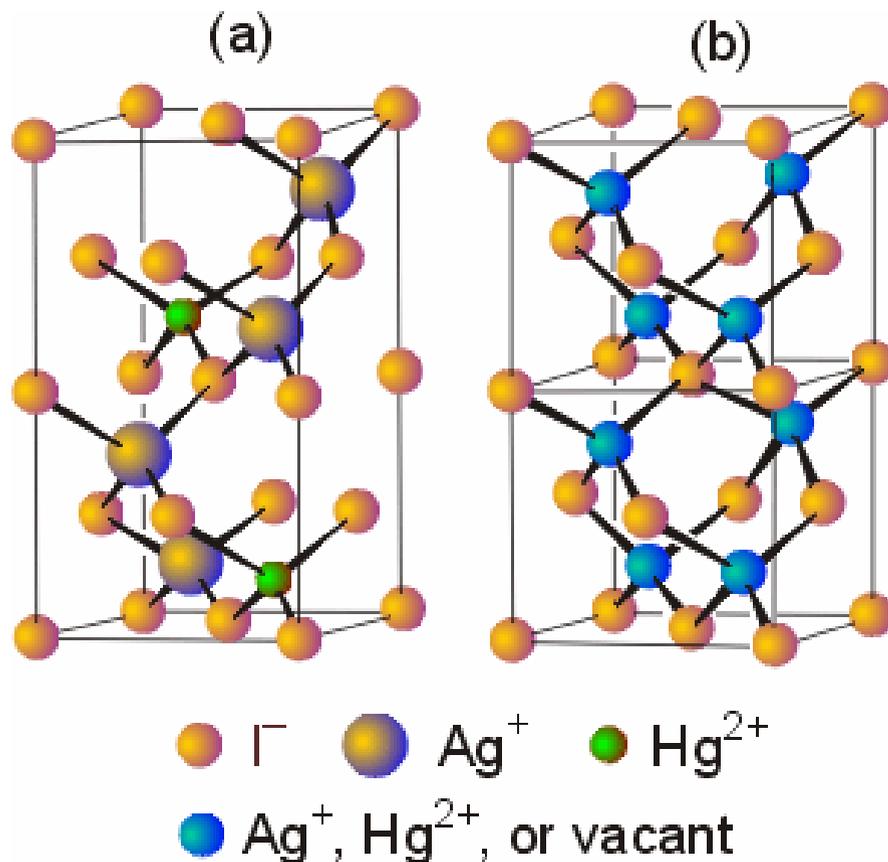


Activation Energy of Ion-Conductor



Arrhenius plot of the logarithm of the diffusion coefficient of the mobile ion

$$D = D_0 \exp(-E_a/RT)$$



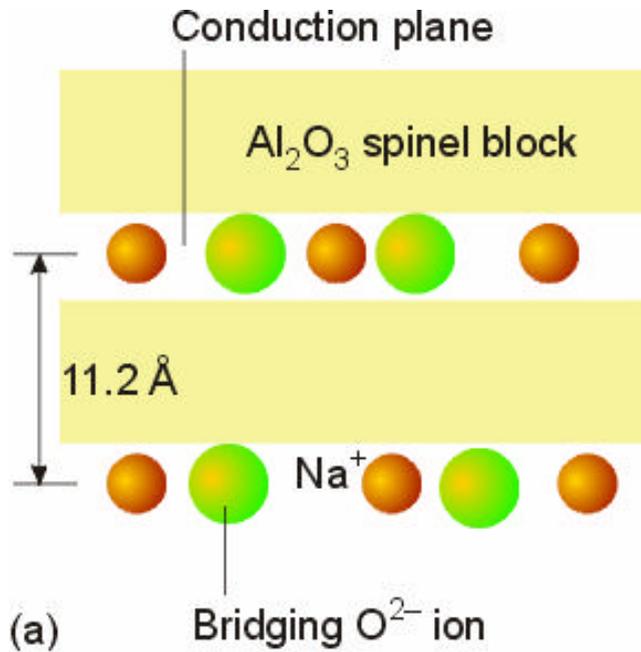
(a) low-temperature order structure

(b) high-temperature disordered structure

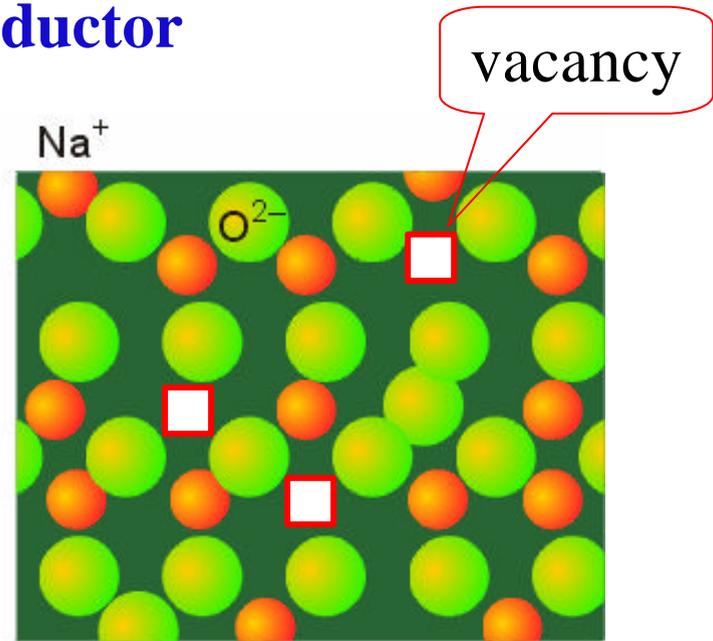
a Ag⁺-conductor

b-Alumina

a Na⁺-conductor

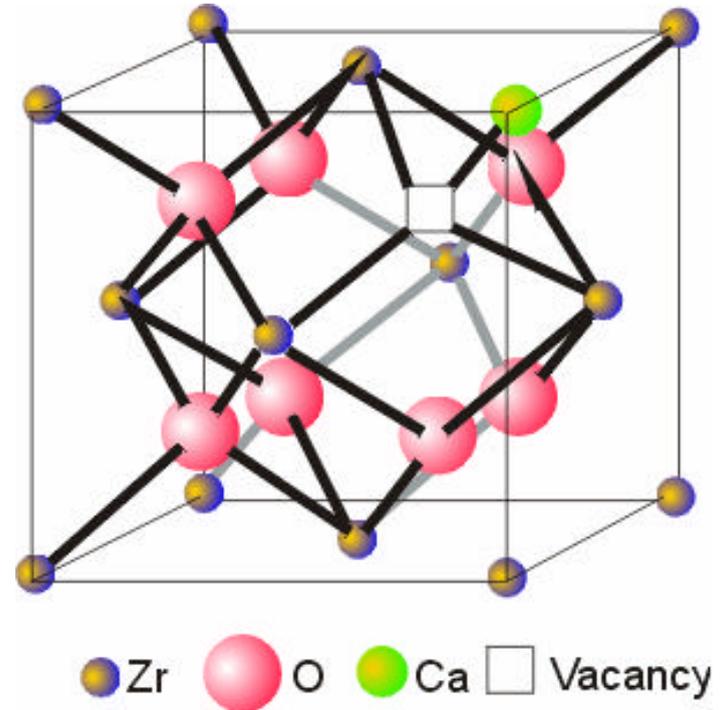
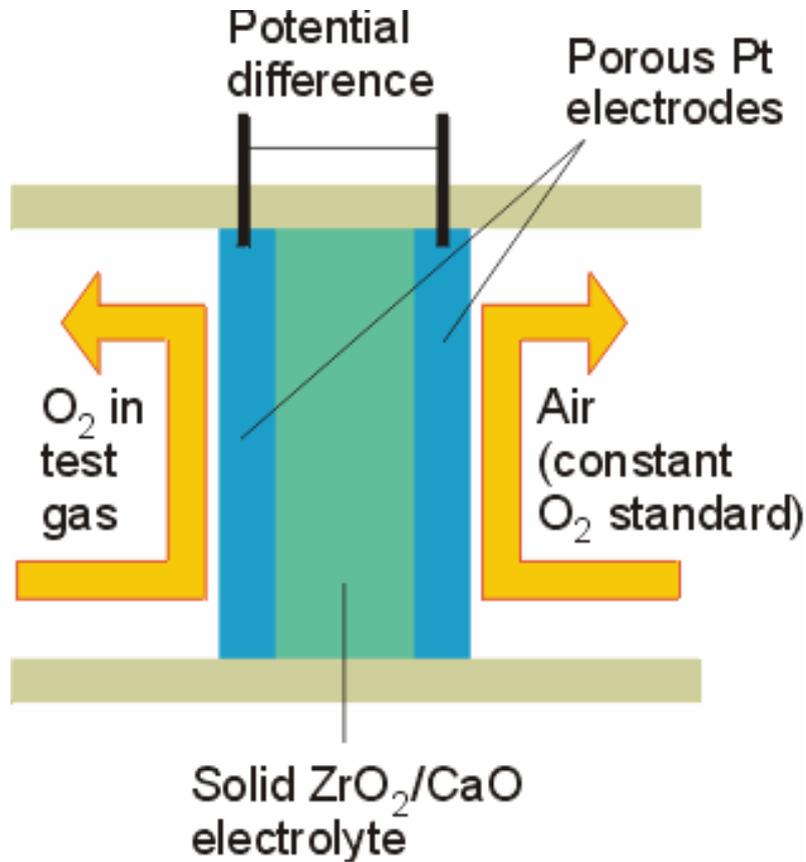


Side view

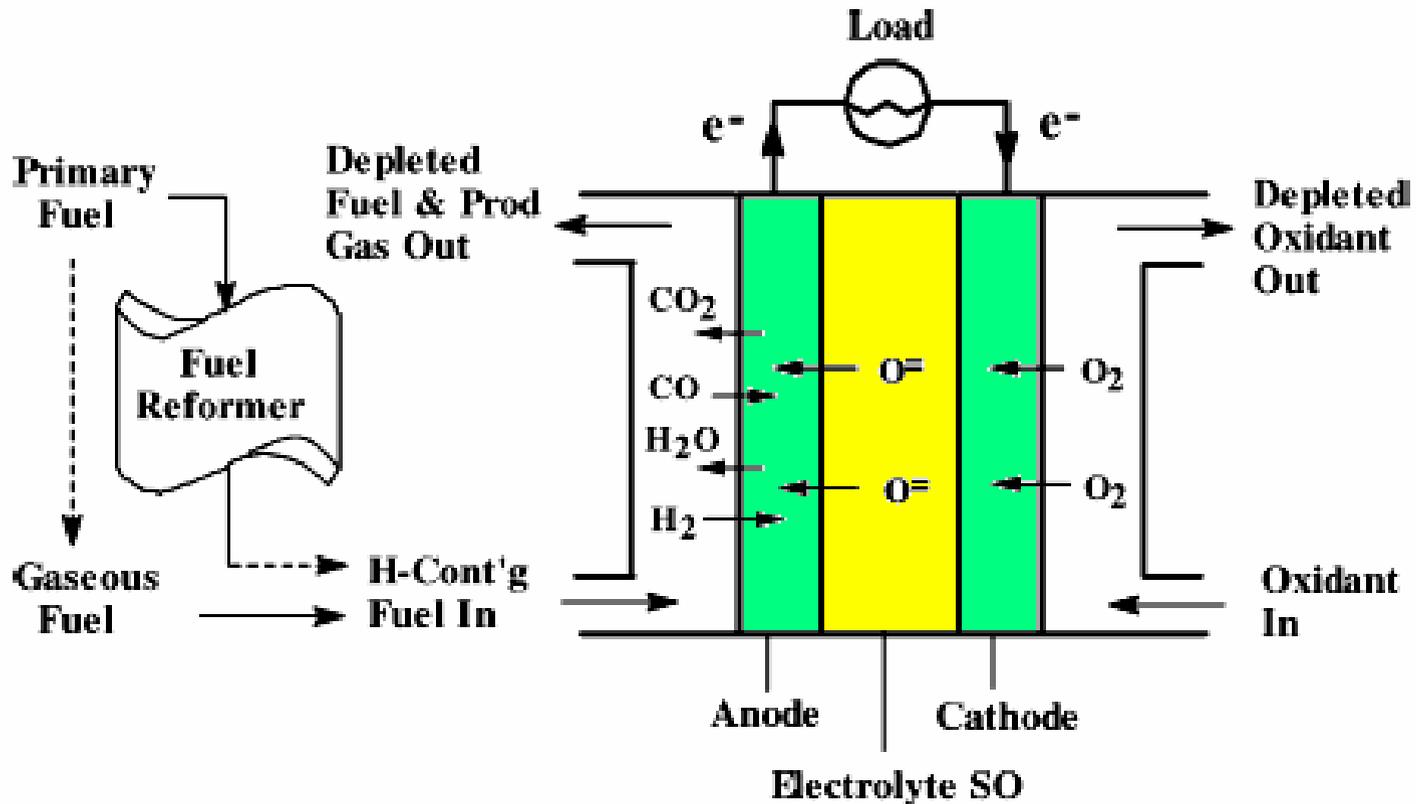


Conducting plane

an O^{2-} -conductor



Solid Oxide Fuel Cell (SOFC)



Anode (Fuel) Reaction:



Cathode (Oxidant) Reaction:



Total Reaction:

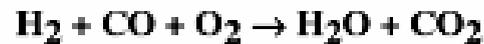
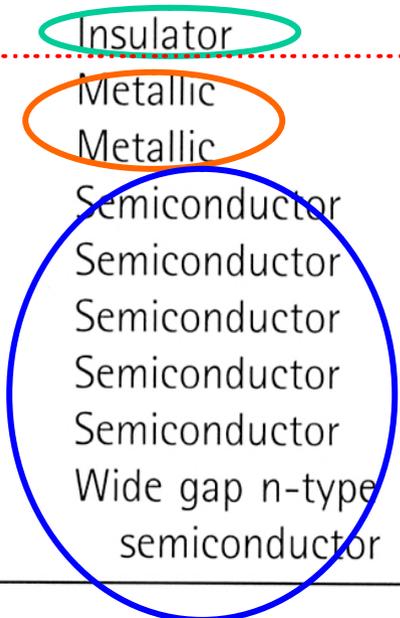


Table 18.2 Monoxides of the Period 4 metals

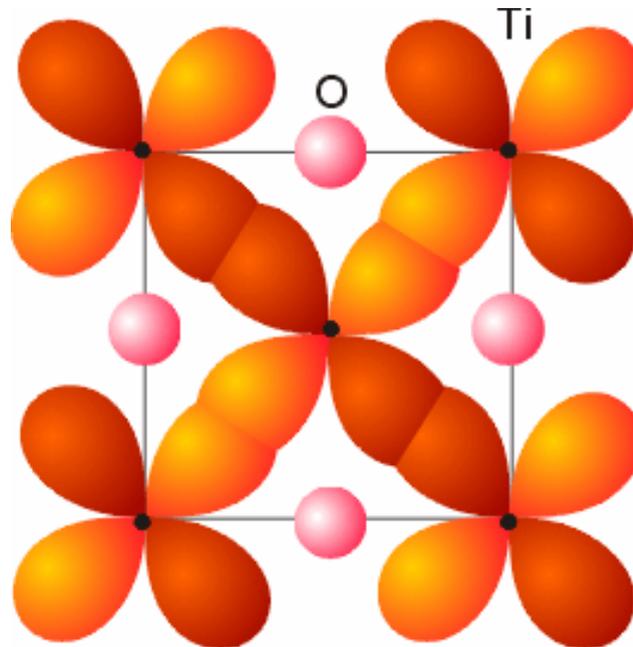
Compound	Structure	Stoichiometry MO_x	Electrical properties
CaO_x	Rock salt	1	Insulator
TiO_x	Rock salt	0.65–1.25	Metallic
VO_x^a	Rock salt	0.79–1.29	Metallic
MnO_x	Rock salt	1–1.15	Semiconductor
FeO_x	Rock salt	1.04–1.17	Semiconductor
CoO_x	Rock salt	1–1.01	Semiconductor
NiO_x	Rock salt	1–1.001	Semiconductor
CuO_x	PtS	1	Semiconductor
ZnO_x	Wurtzite	Slight Zn excess	Wide gap n-type semiconductor

^aAt low temperatures, VO adopts a less symmetrical structure.

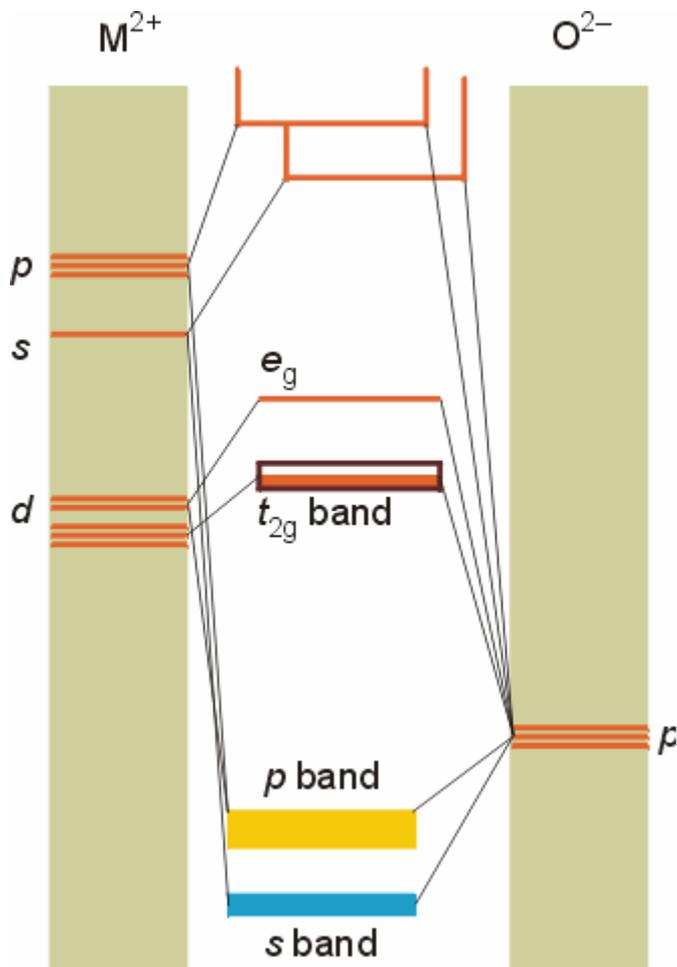
E_g

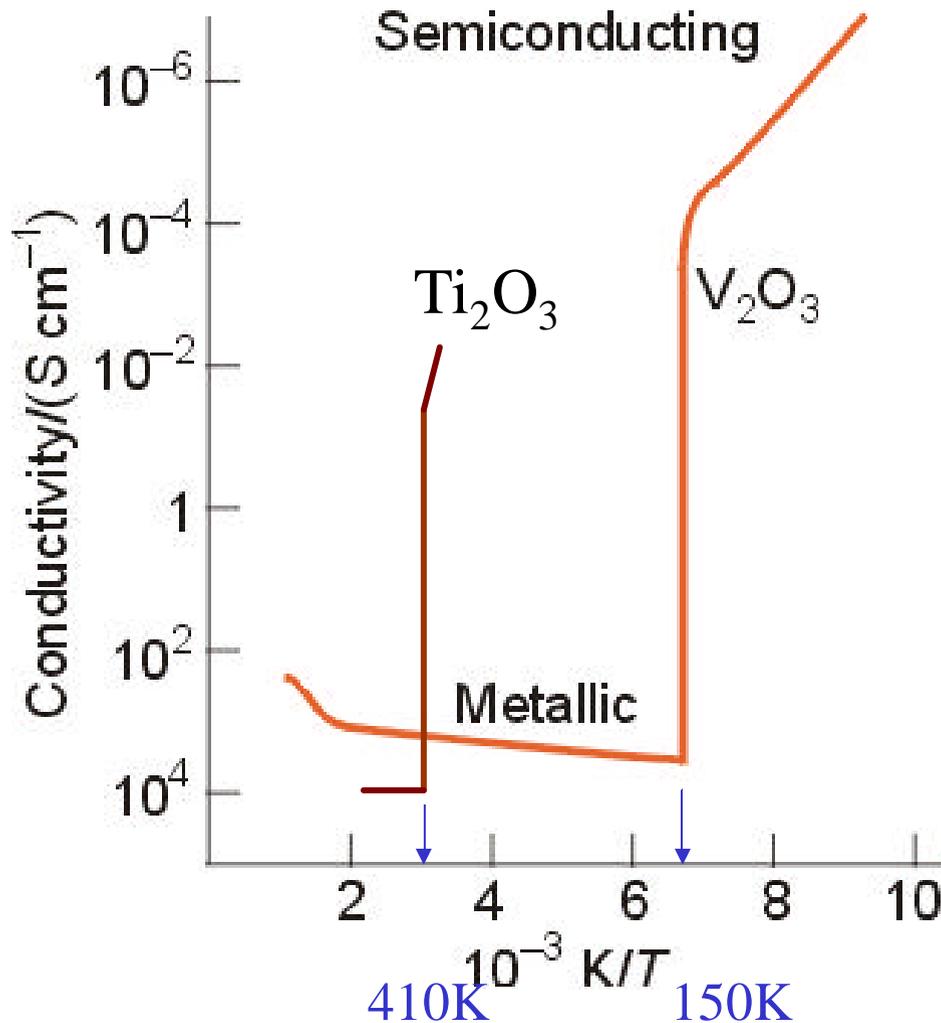


Overlapping of d-Orbitals of Early Transition Metal Elements in the Oxide Structures

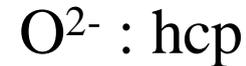


Energy Level Diagram of Early Transition Metal Elements in the Oxide Structures





Corundum structure



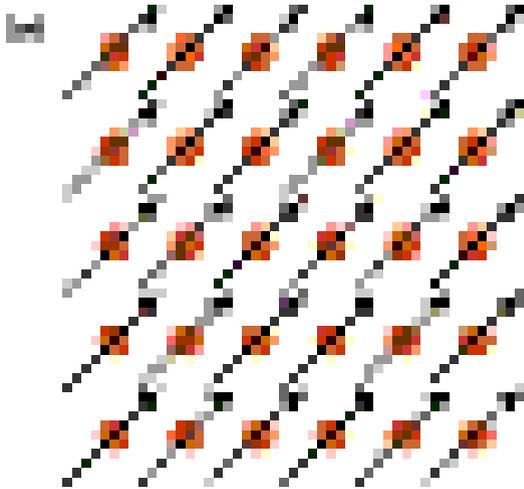
Cr(III) doped $\alpha\text{-Al}_2\text{O}_3$

\Rightarrow **ruby**

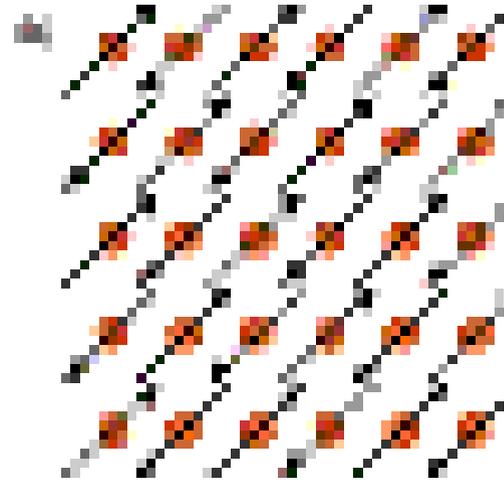
Cr_2O_3 , Fe_2O_3 insulators

& **antiferromagnetic**

Magnetic Properties of Materials with Long-Range Ordered Spins



Ferromagnetic



Antiferromagnetic



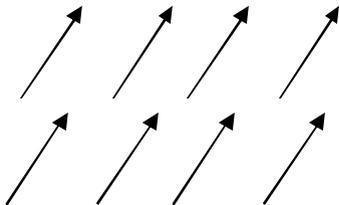
J: energy of interaction of the spins on different ions

If $J < kT$ paramagnetic

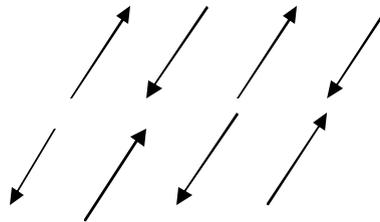
Property of individual ion

If $J > kT \Rightarrow$ {
Ferromagnetic
Antiferromagnetic

Cooperative magnetism



Ferromagnetic



Antiferromagnetic

Molar magnetic susceptibility, χ

In a magnetic field, H , the magnetic induction, B , in the material is

$B = H + 4\pi I$, where I = magnetic moment/ volume

magnetic permeability, $P = B/H = 1 + (4\pi I)/H = 1 + 4\pi \kappa$

magnetic susceptibility, $\kappa = I/H$

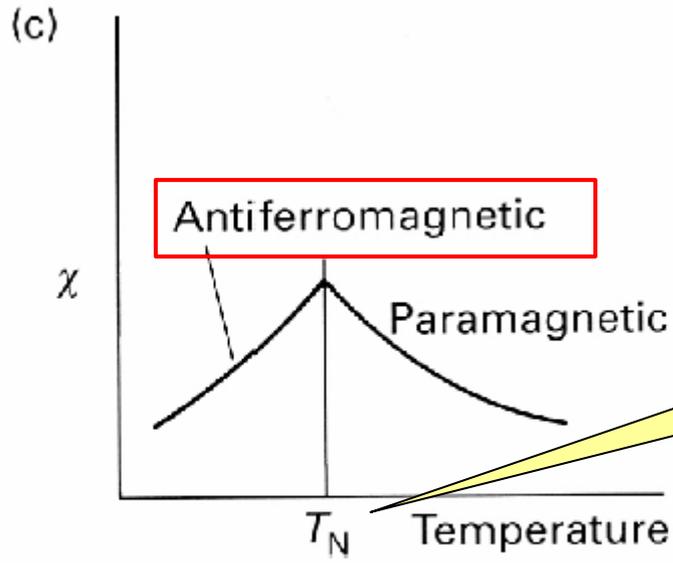
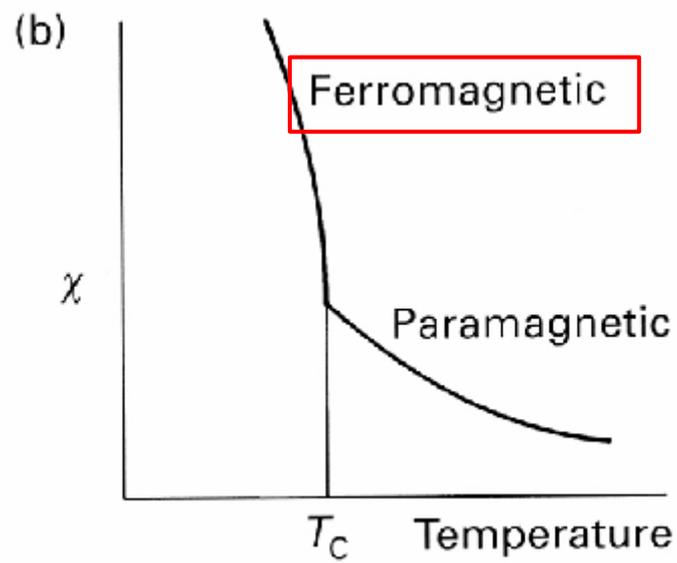
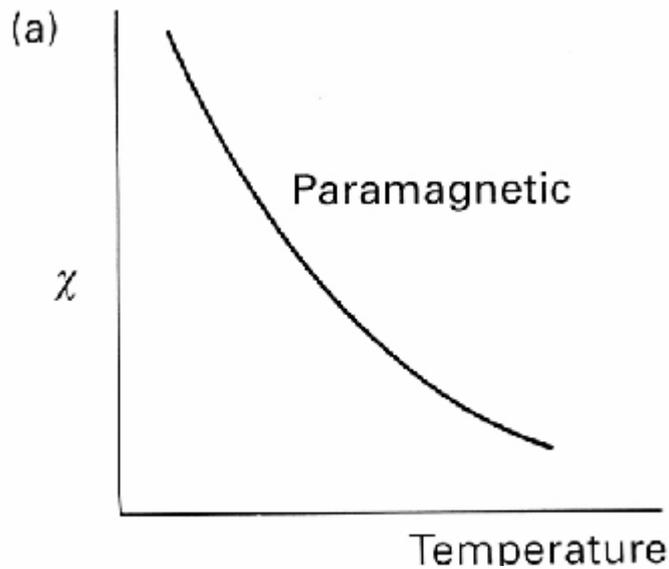
Molar magnetic susceptibility

$c = \kappa F/d$, where F = formula weight

d = density of the material

Curie-Weiss Law: $c = C/(T + q)$,

C : Curie constant; q : Weiss constant



Curie temperature

Neel temperature

Cooperative magnetism

B18.3 Comparison of the temperature dependence of paramagnetic, ferromagnetic, and antiferromagnetic substances.

Intercalation Reactions

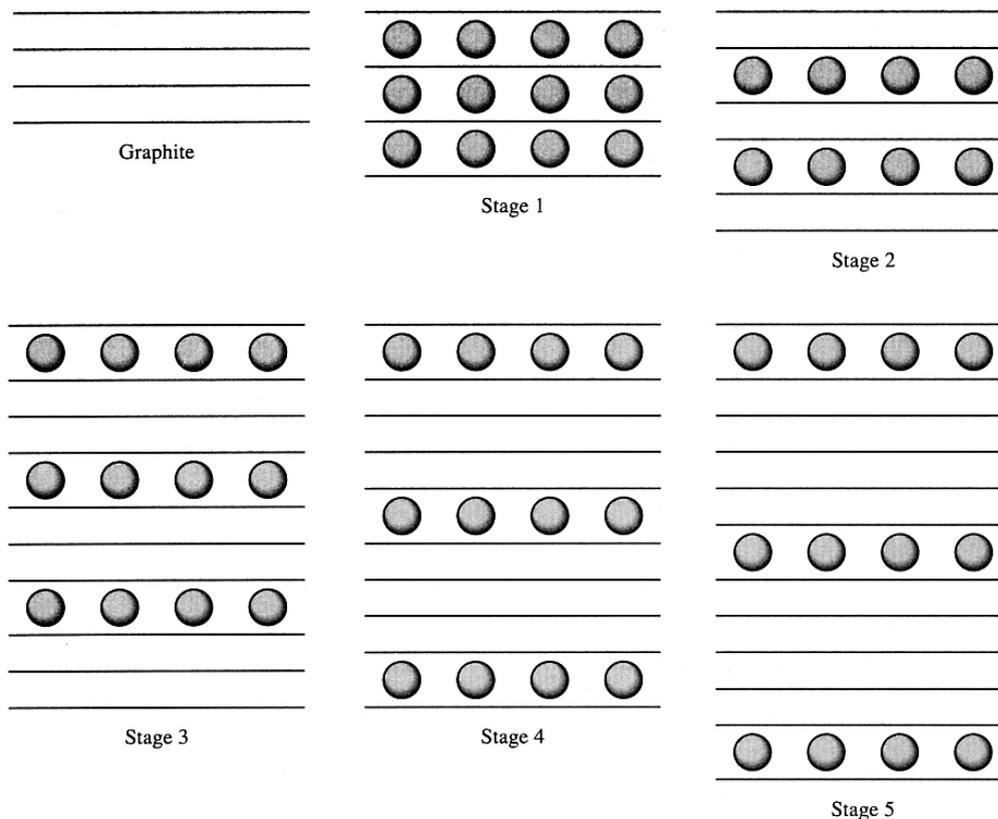
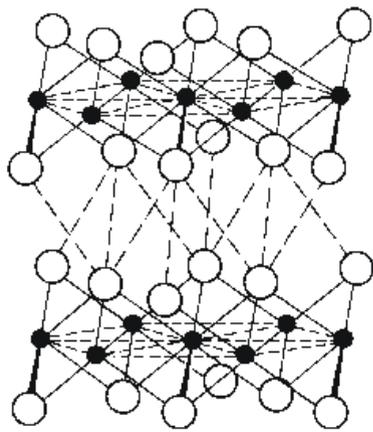
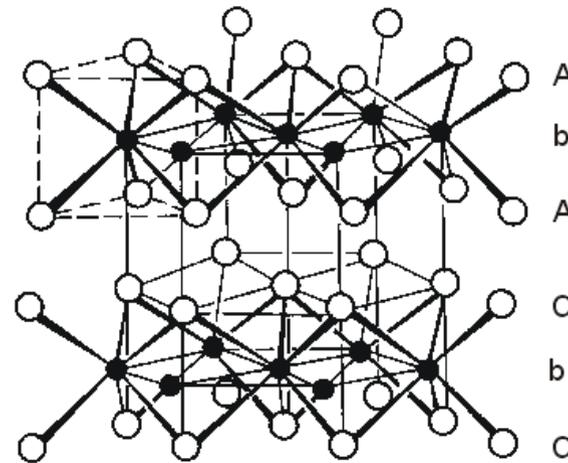


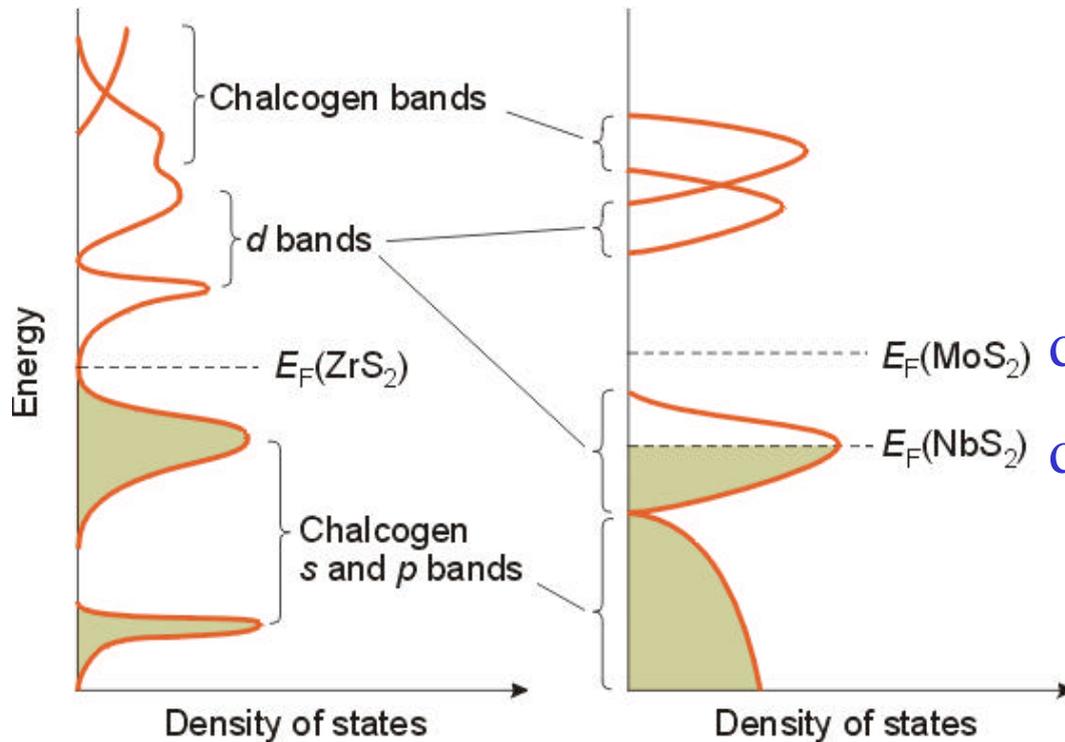
Figure 6.33 Graphite intercalation compounds indicating different staging.



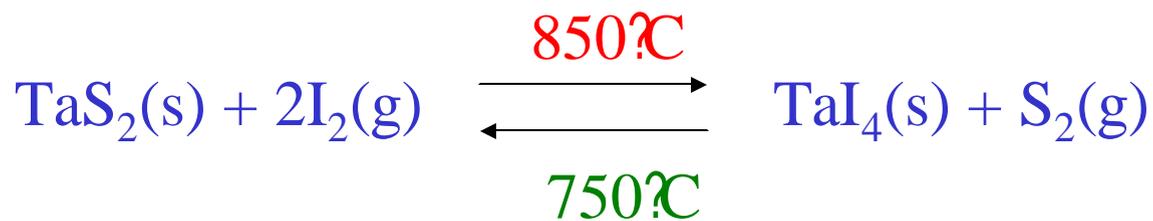
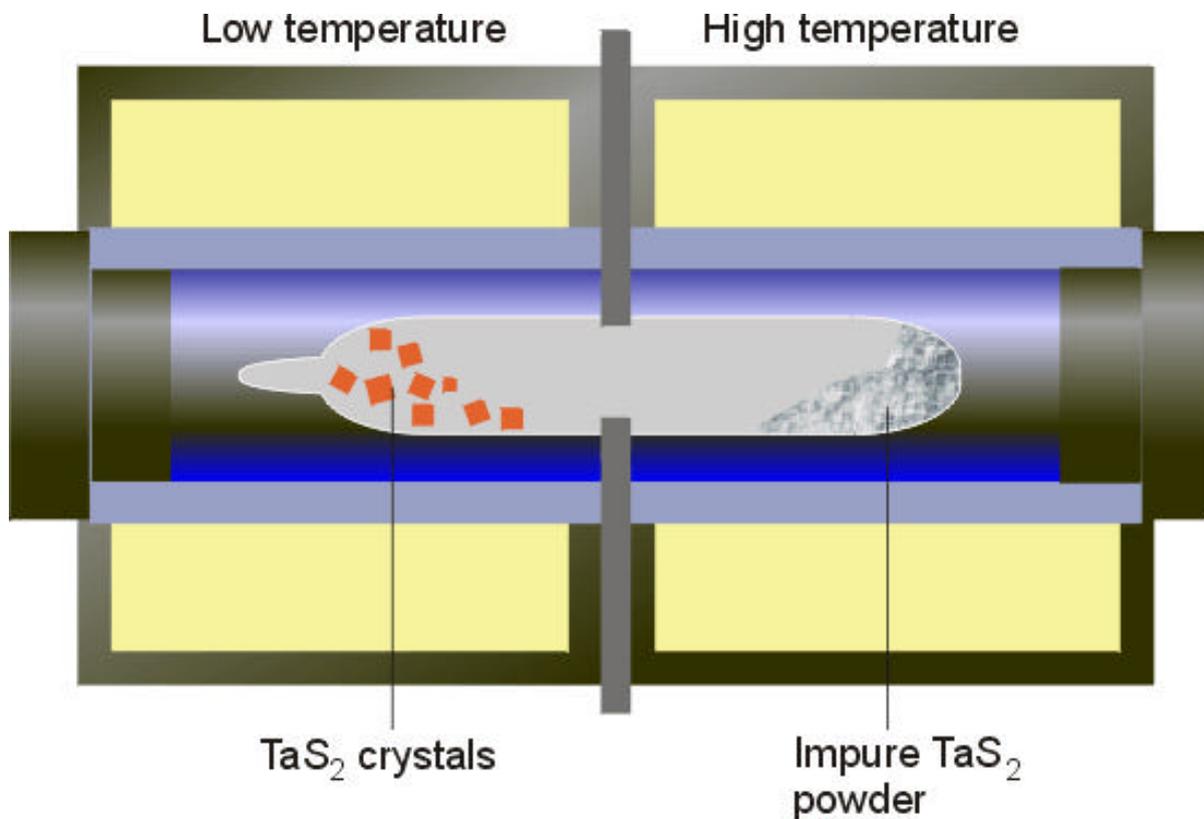
(a) ● Ta ○ S



(b) ● Mo ○ S

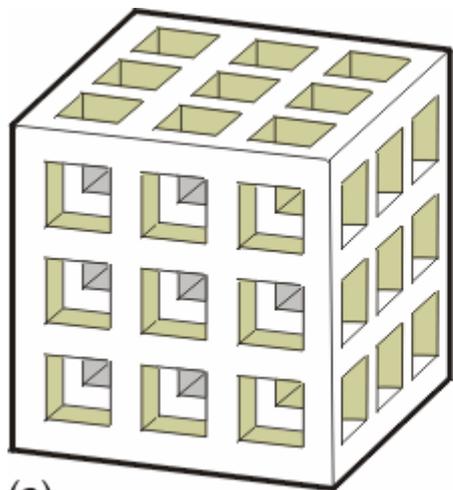


Chemical vapor transport crystal growth

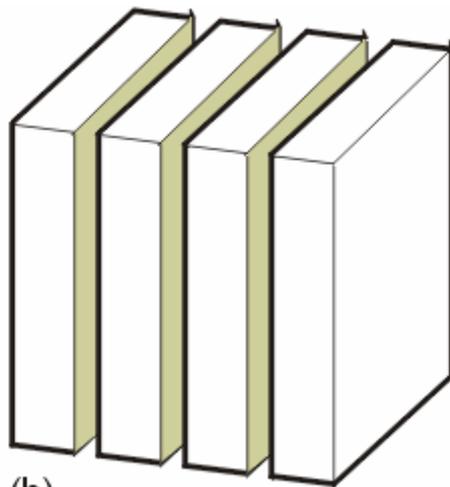


18.32

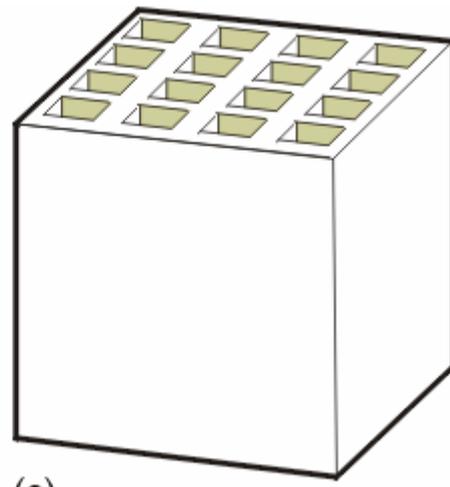
Hosts for Intercalation



(a)



(b)

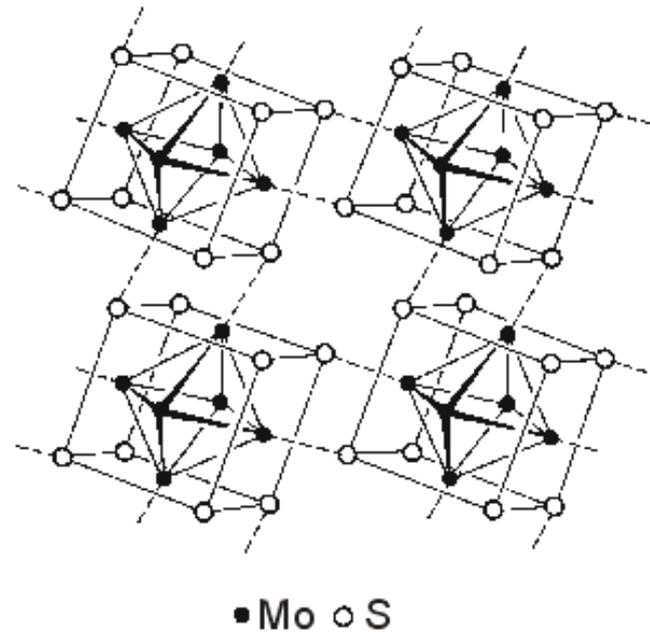
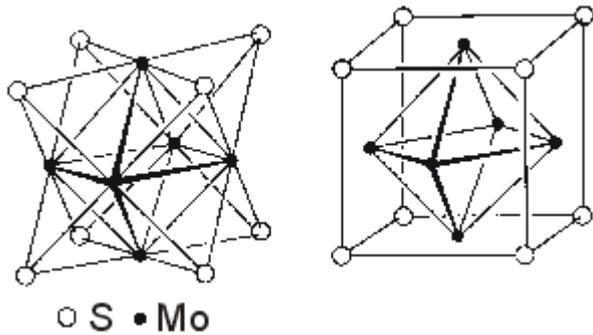


(c)

Table 18.6 Some three-dimensional intercalation compounds

Phase	Composition (x)
$\text{Li}_x[\text{Mo}_6\text{S}_8]$	2.4 to 0.6
$\text{Na}_x[\text{Mo}_6\text{S}_8]$	3.6
$\text{Ni}_x[\text{Mo}_6\text{Se}_8]$	1.8
H_xWO_3	0.6
Li_xWO_3	0.6
Li_xNiPS_3	0 to 1.5

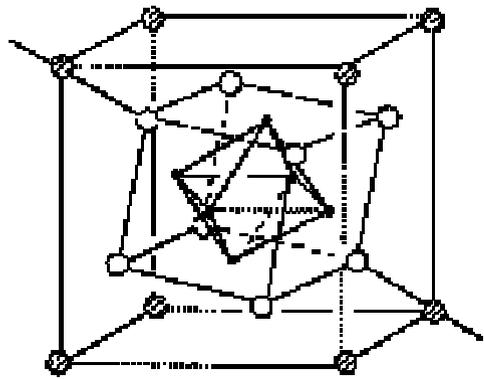
Chevrel phases- three-dimensional intercalation



M= Li, Mn, Fe, Cd, Pb

PbMo_6S_8 superconductor at 14K

18.38-39



- M
- X
- Mo

Figure 2. Elementary cell of rhombohedral structure

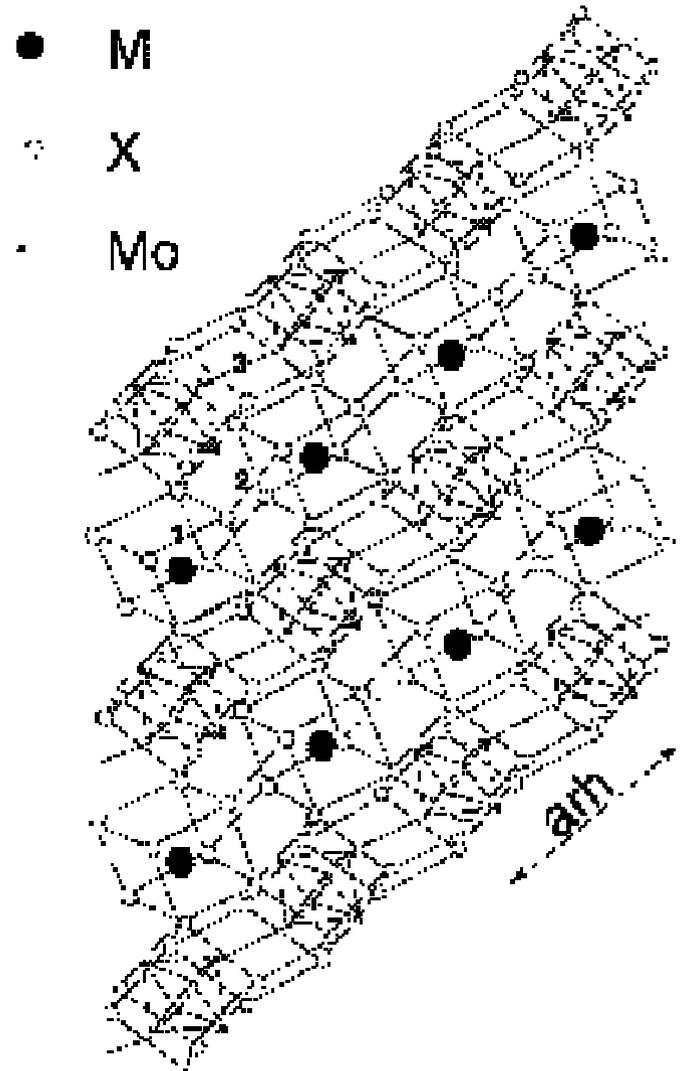
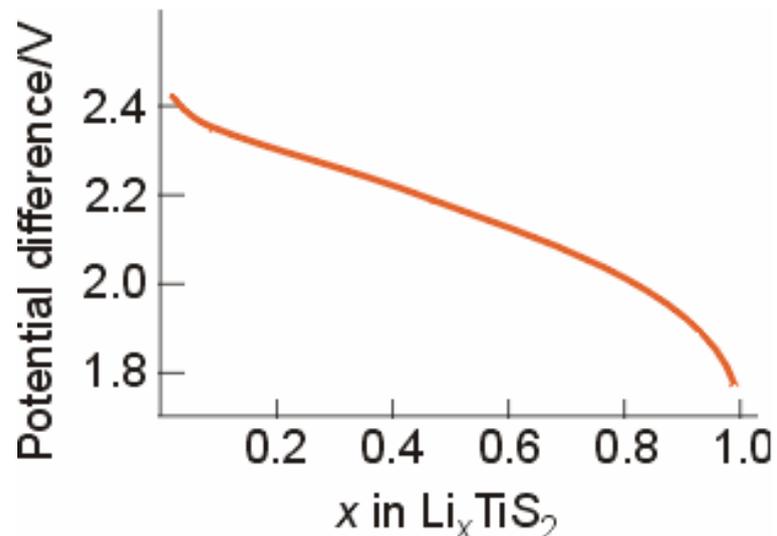
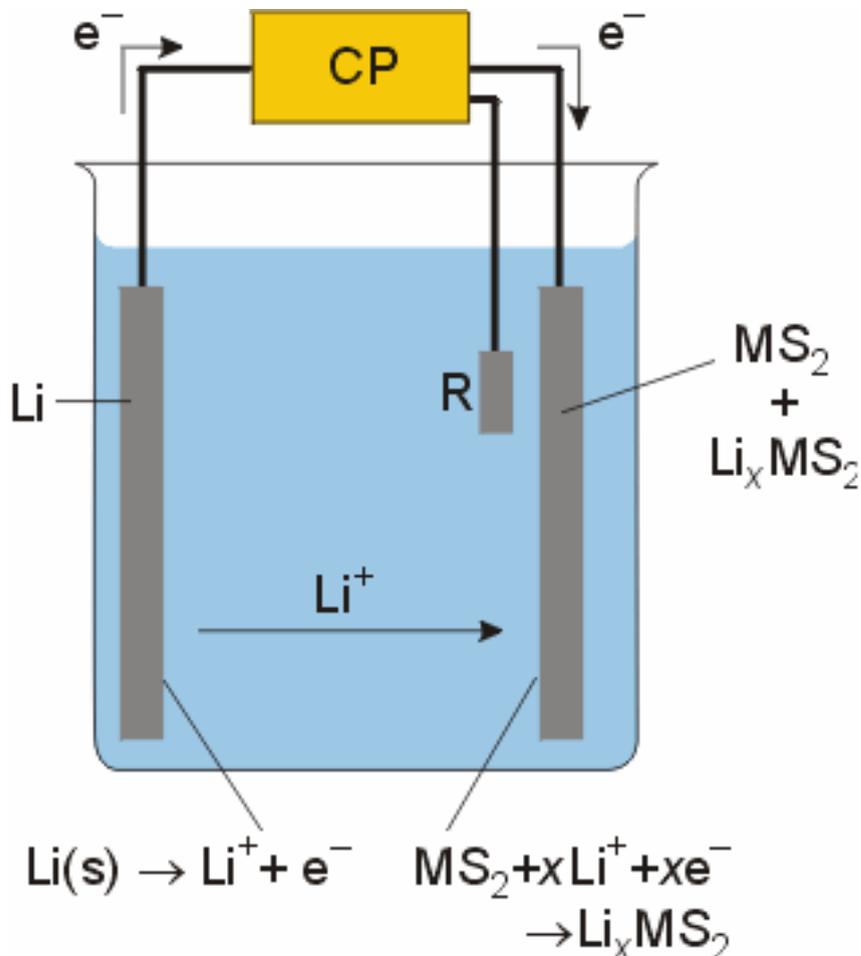
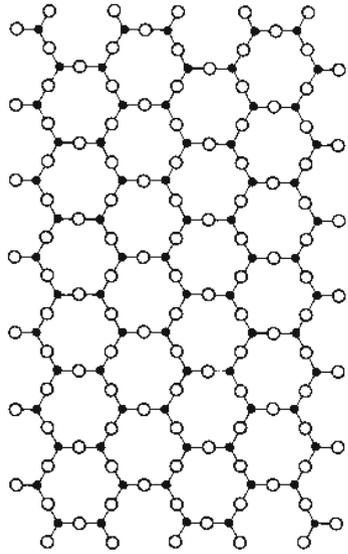


Figure 3. Projection on the hexagonal $(11\bar{2}0)$ plane. The three types of cavity are represented. Large cation are located in site 1.

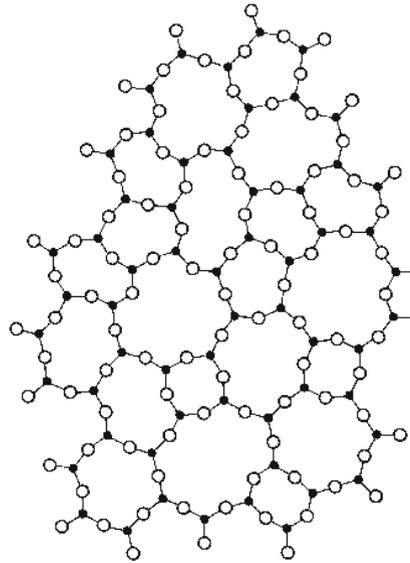
Intercalation Reaction in Lithium Battery



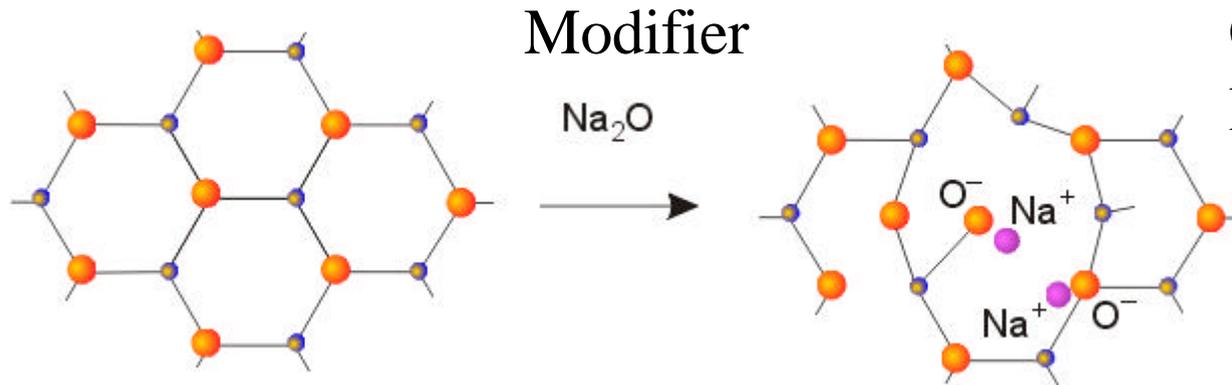
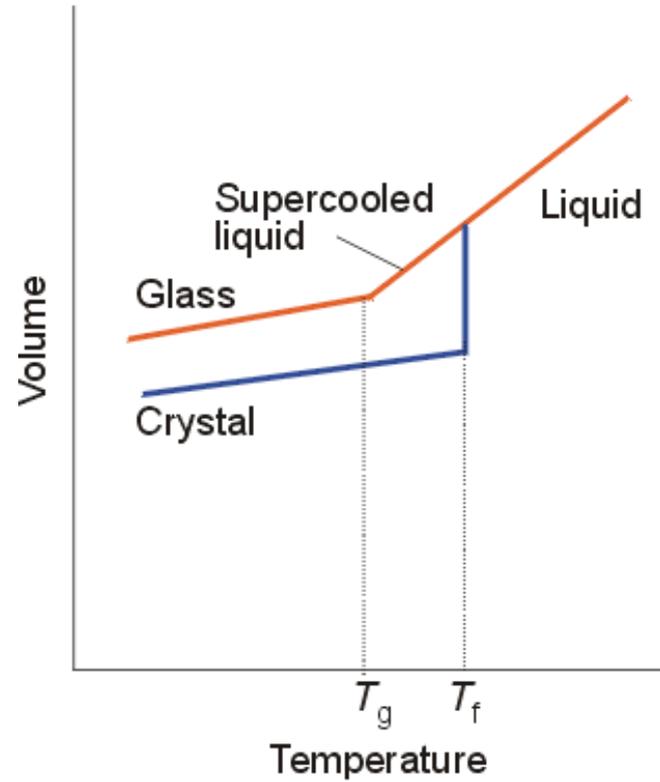
Glass formation



(a)



(b)



Modifier

Glass of lower m.p.

Synthesis of Solid (Oxide) Materials by “Sol-Gel Method”

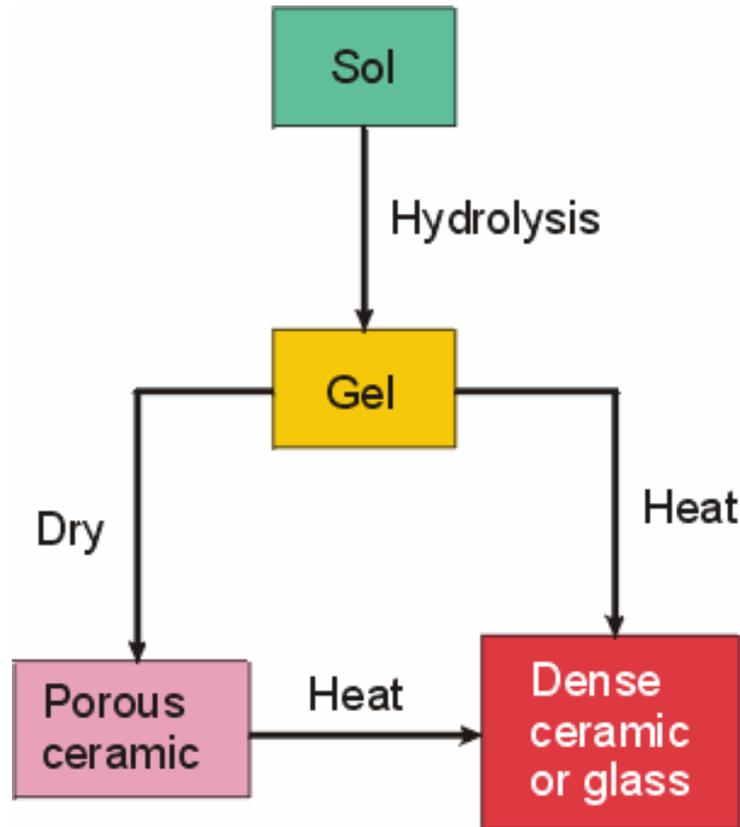
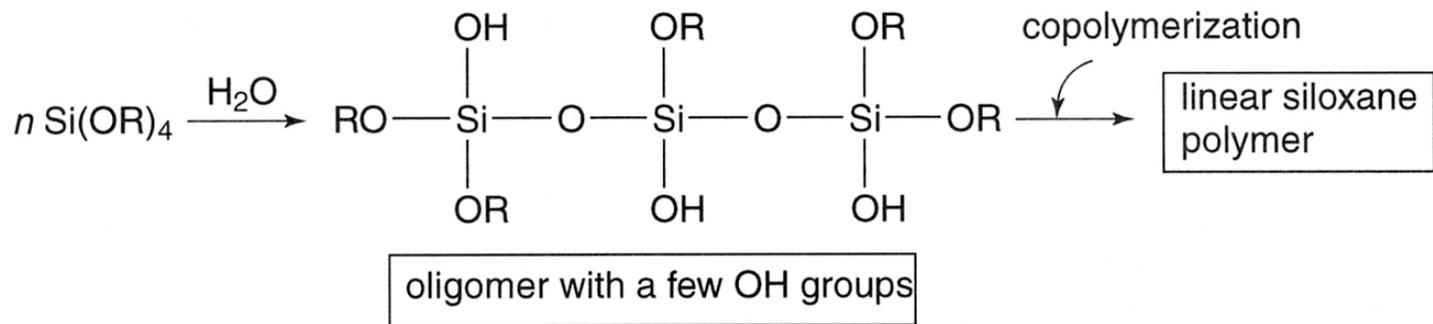


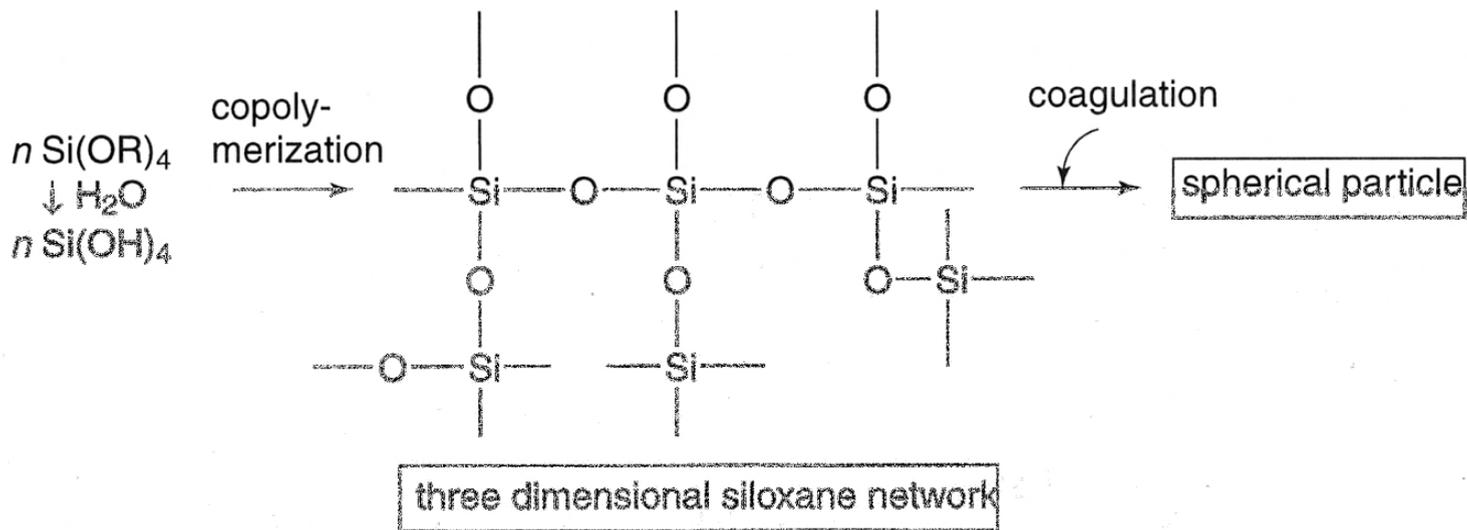
Chart18.1

Table 7.15 Some Metal Alkoxides Used for Sol-gel Processing of Ceramics

		Alkoxide
Single cation alkoxides		
I A (1) group	Li, Na	LiOCH ₃ (s), NaOCH ₃ (s)
I B (11) grp.	Cu	Cu(OCH ₃) ₂ (s)
II A (2) grp.	Ca, Sr, Ba	Ca(OCH ₃) ₂ (s) Sr(OC ₂ H ₅) ₂ , Ba(OC ₂ H ₅) ₂ (s)
II B (12) grp.	Zn	Zn(OC ₂ H ₅) ₂ (s)
III A (3) grp.	B, Al, Ga	B(OCH ₃) ₃ (l) Al(<i>i</i> - OC ₃ H ₇) ₃ (s) Ga(OC ₂ H ₅) ₃ (s)
III B (13) grp.	Y	Y(OC ₄ H ₉) ₃
IV A (4) grp.	Si, Ge	Si(OC ₂ H ₅) ₄ (l) Ge(OC ₂ H ₅) ₄ (l)
IV B (14) grp.	Pb	Pb(OC ₄ H ₉) ₄ (s)
V A (5) grp.	P, Sb	P(OCH ₃) ₃ (l) Sb(OC ₂ H ₅) ₃ (l)
V B (15) grp.	V, Ta	VO(OC ₂ H ₅) ₃ (l) Ta(OC ₃ H ₇) ₅ (l)
VI B (16) grp.	W	W(OC ₂ H ₅) ₆ (s)
lanthanide	La, Nd	La(OC ₃ H ₇) ₃ (s) Nd(OC ₂ H ₅) ₃ (s)
Alkoxides with various alkoxy groups		
	Si	Si(OCH ₃) ₄ (l) Si(OC ₂ H ₅) ₄ (l) Si(<i>i</i> - OC ₃ H ₇) ₄ (l) Si(<i>t</i> - OC ₄ H ₉) ₄
	Ti	Ti(OCH ₃) ₄ (s) Ti(OC ₂ H ₅) ₄ (l) Ti(<i>i</i> - OC ₃ H ₇) ₄ (l) Ti(OC ₄ H ₉) ₄ (l)
	Zr	Zr(OCH ₃) ₄ (s) Zr(OC ₂ H ₅) ₄ (s) Zr(OC ₃ H ₇) ₄ (s) Zr(OC ₄ H ₉) ₄ (s)
	Al	Al(OCH ₃) ₃ (s) Al(OC ₂ H ₅) ₃ (s) Al(<i>i</i> - OC ₃ H ₇) ₃ (s) Al(OC ₄ H ₉) ₃ (s)
Double cation alkoxides		
	La-Al	La[Al(<i>i</i> - OC ₃ H ₇) ₄] ₃
	Mg-Al	Mg[Al(<i>i</i> - OC ₃ H ₇) ₄] ₂ , Mg[Al(<i>s</i> - OC ₄ H ₉) ₄] ₂
	Ni-Al	Ni[Al(<i>i</i> - OC ₃ H ₇) ₄] ₂
	Zr-Al	(C ₃ H ₇ O) ₂ Zr[Al(OC ₃ H ₇) ₄] ₂
	Ba-Zr	Ba[Zr ₂ (OC ₂ H ₅) ₉] ₂

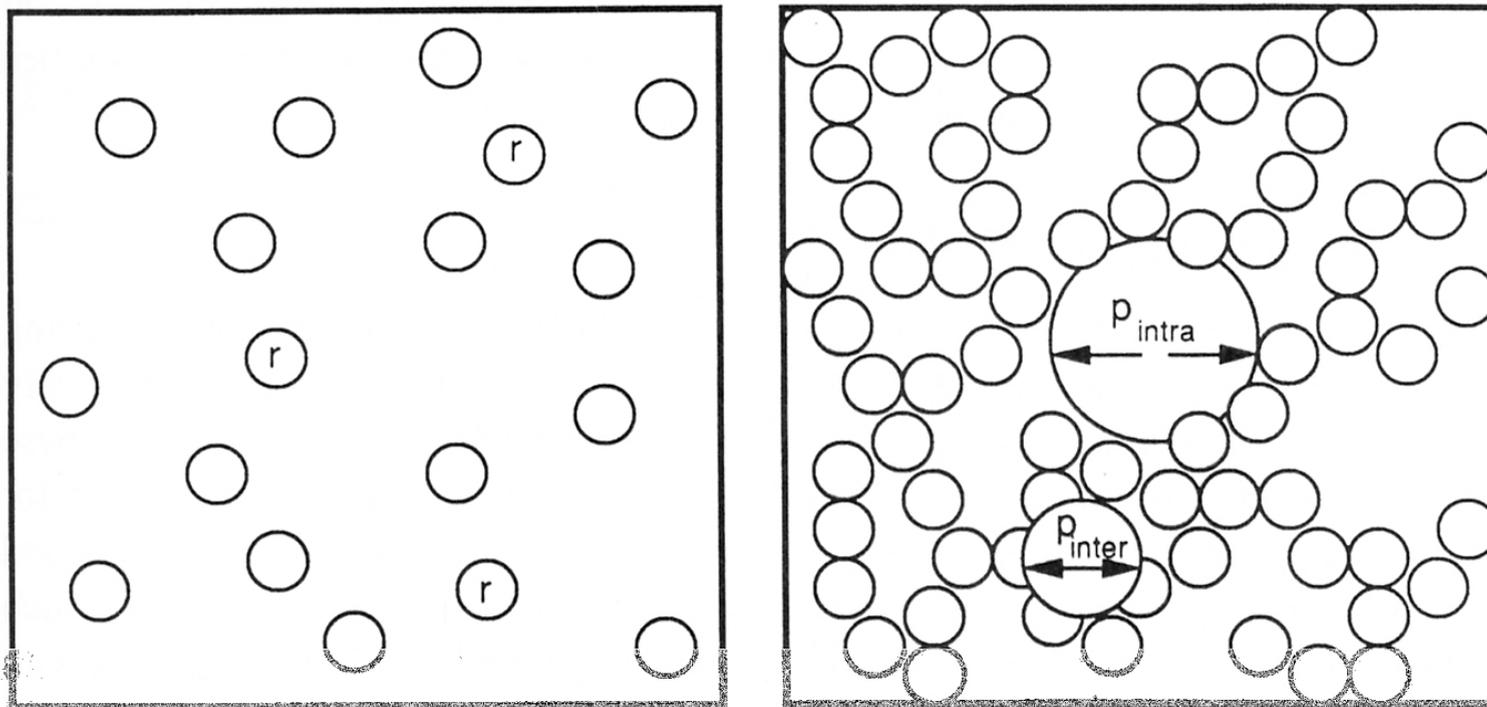


(a)



(b)

Figure 7.53 Hydrolysis and polymerization of a generic alkoxide Si(OR)_4 involving both (a) acid and (b) basic routes. Reprinted, by permission, from H. Yanagida, K. Koumoto, and M. Miyayama, *The Chemistry of Ceramics*, p. 148. Copyright © 1996 by John Wiley & Sons, Inc.



SOL

$$r = \sim 2-3 \text{ nm}$$

GEL

Micropore

$$p_{\text{inter}} = \sim 7 \text{ nm}$$

Pore

$$p_{\text{intra}} = \sim 12 \text{ nm}$$

TRANSITION

Figure 7.3. Schematic diagram of the linked particles of a dried gel following the colloidal route.

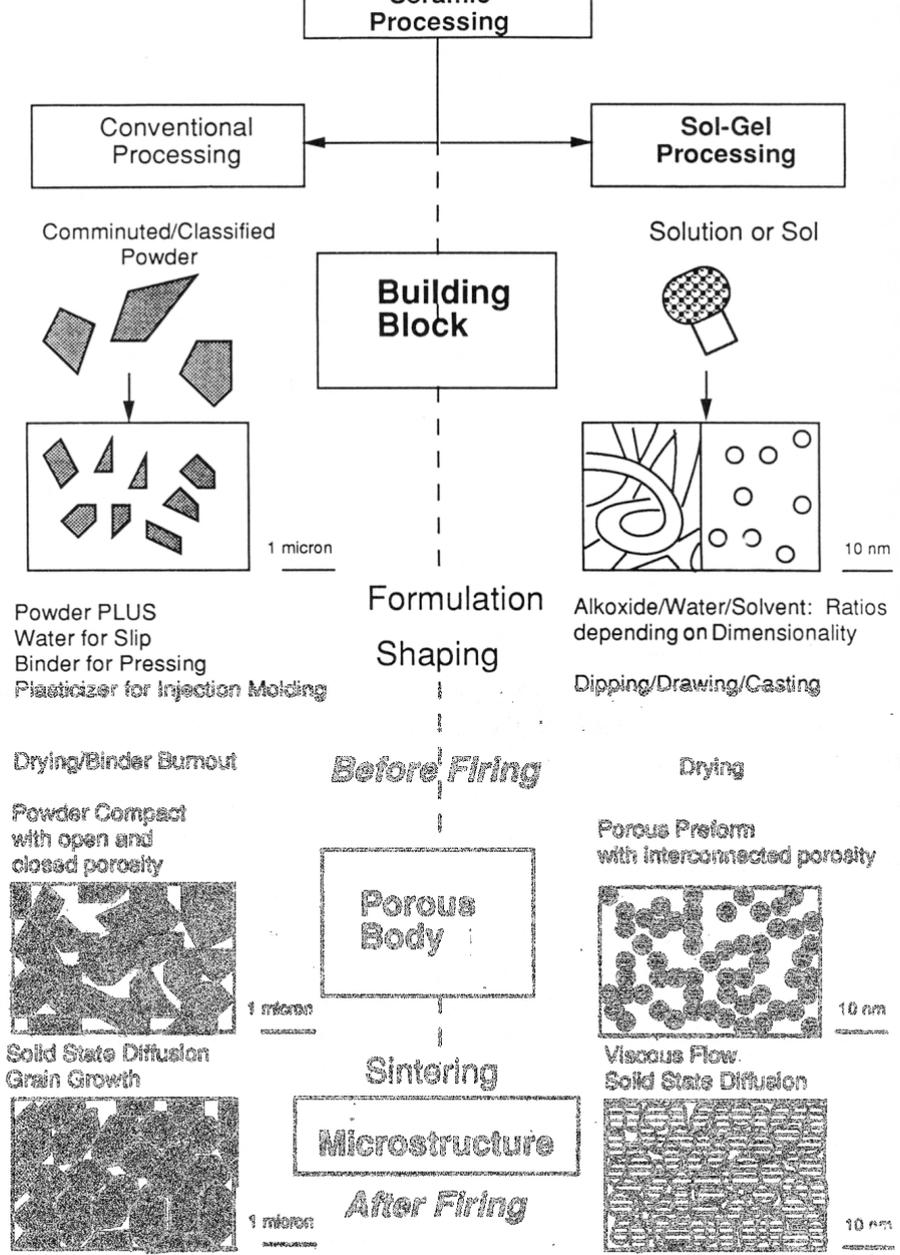
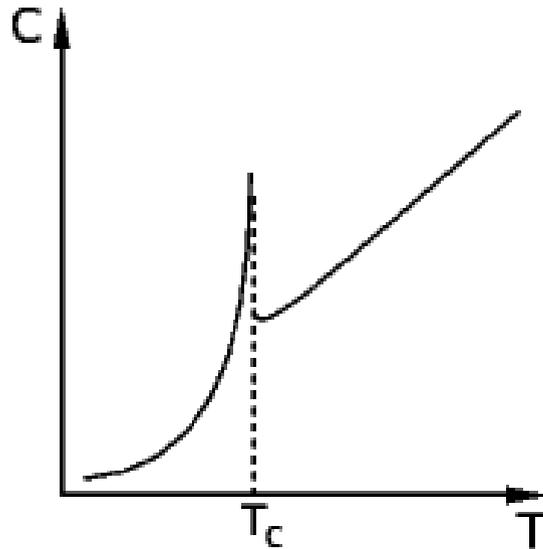


Figure 7.5. Comparison of steps in conventional processing of ceramics with those in

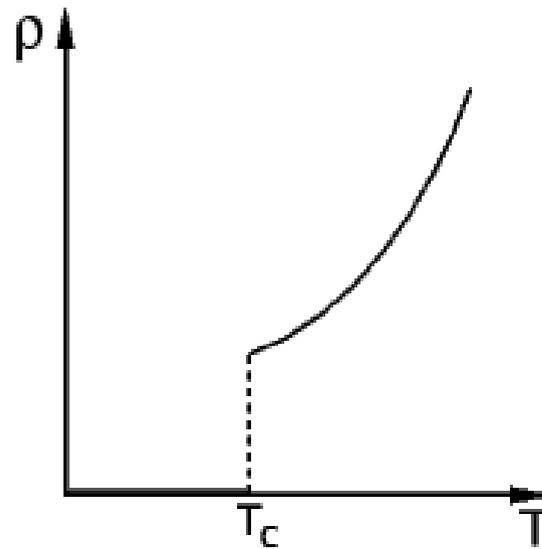
Superconductor

- Zero electrical resistance at temperatures below critical temperature
- Superconducting phase transition
- Meissner effect

Heat capacity



resistance





Meissner effect with high temp. superconductor
Photo from Pacific Northwest National Laboratory

											H						He
Li	Be											B	C	N	O	F	Ne
Na	Mg											Al	Si	P	S	Cl	Ar
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
Cs	Ba	La-Lu	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
Fr	Ra	Ac-Lr															

La	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu
Ac	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr

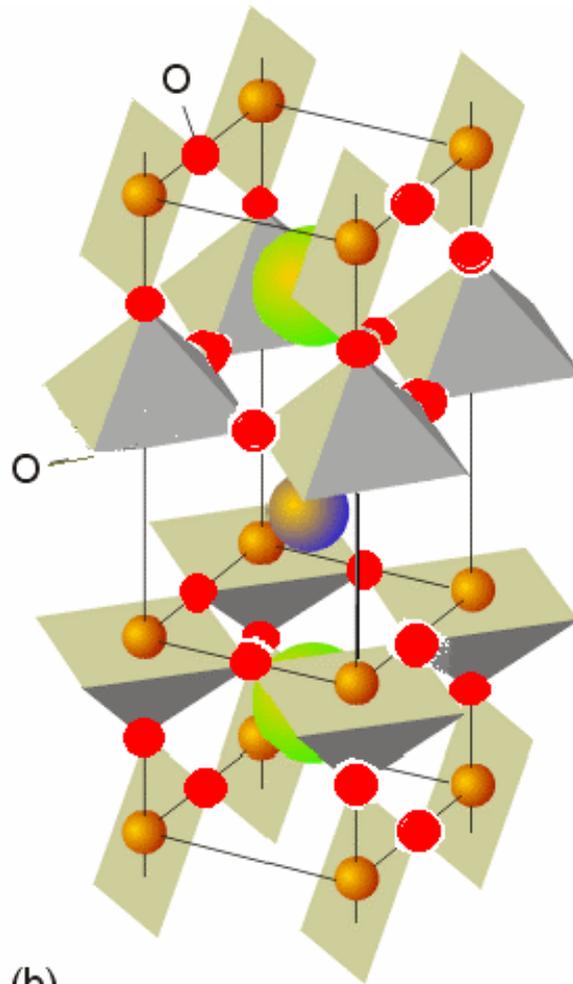
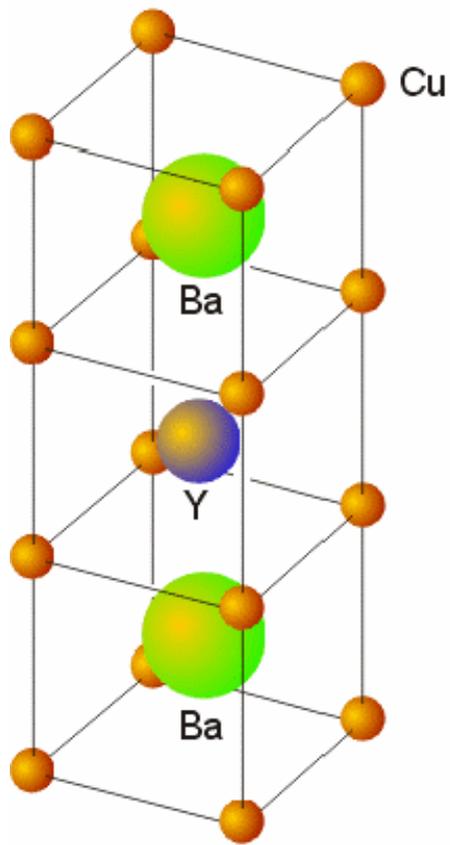
-  Superconducting elements
-  Superconducting elements under pressure

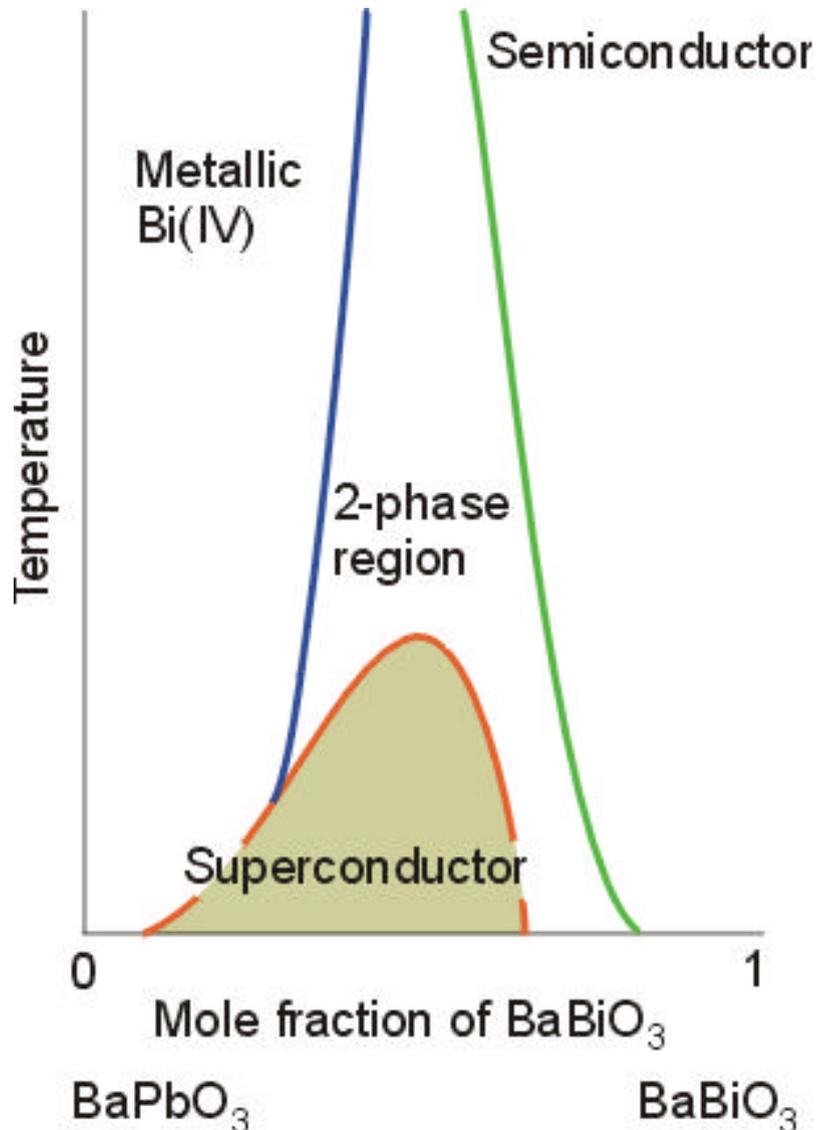
-  Superconducting elements in thin films

Table 18.4 Some superconductors

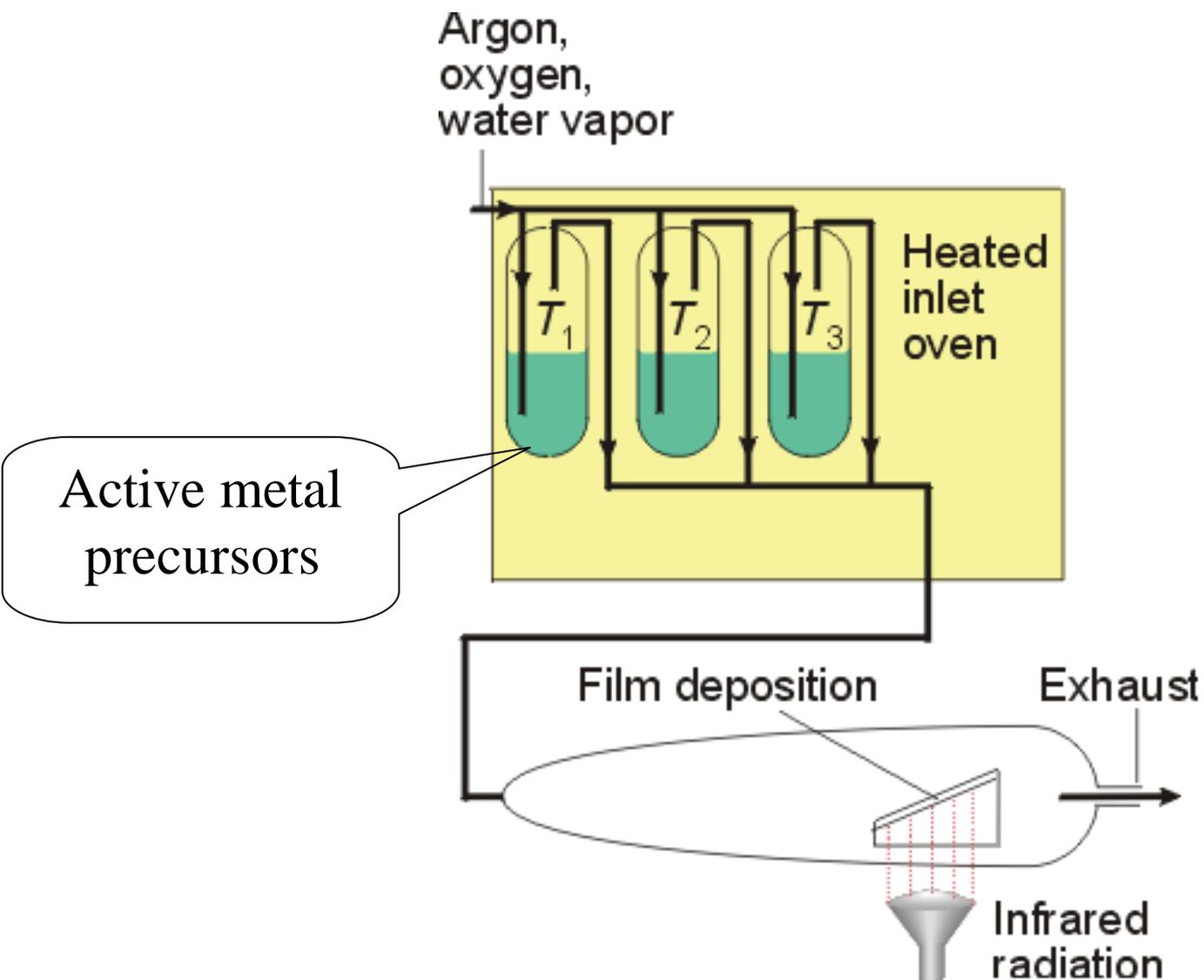
Elements	T_c /K	Compounds	T_c /K
Zn	0.88	Nb ₃ Ge	23.2
Cd	0.56	Nb ₃ Sn	18.0
Hg	4.15	LiTiO ₄	13
Pb	7.19	K _{0.4} Ba _{0.6} BiO ₃	29.8
Nb	9.50	YBa ₂ Cu ₃ O ₇	95
		Tl ₂ Ba ₂ Ca ₂ Cu ₃ O ₁₀	122

High temperature
superconductor





Superconductivity is usually observed for materials that are not highly conducting above the critical temperature.



Chemical vapor deposition (CVD) of superconductor thin film.
18.28

Buckyball

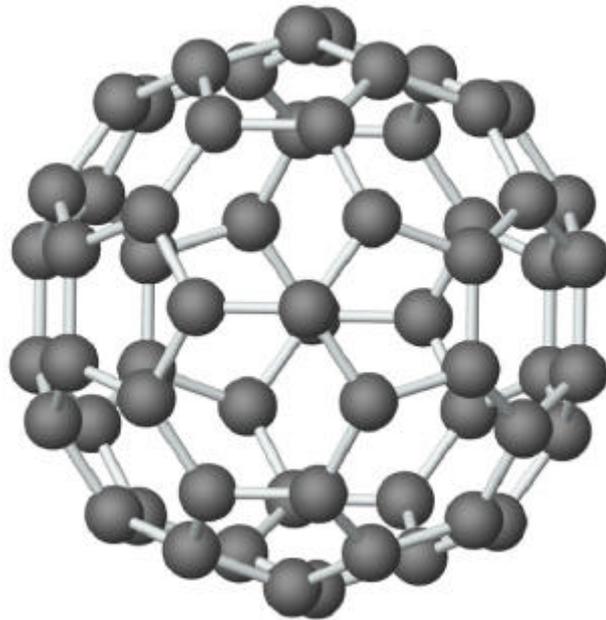


Table 6.3 Physical constants for C₆₀^b

<i>Quantity</i>	<i>Value</i>	<i>Quantity</i>	<i>Value</i>
Average C—C distance	144 pm	Band gap (HOMO–LUMO)	<u>1.7 eV</u>
fcc Lattice constant	1417 pm	Binding energy per atom	7.40 eV
C ₆₀ mean ball diameter	683 pm	Debye temperature	185 K
C ₆₀ ball outer diameter ^a	1018 pm	Thermal conductivity (300 K)	0.4 W/mK
C ₆₀ ball inner diameter ^a	348 pm	Phonon mean free path	5000 pm
Tetrahedral site radius	112 pm	Static dielectric constant	4.0–4.5
Octahedral site radius	207 pm	Velocity of sound v_s	2.1×10^5 cm/sec
Electron affinity (pristine C ₆₀)	2.65 eV	Structural phase transitions	255 K, 165 K
Ionization energy (first)	7.58 eV	Mass density	1.72 g/cm ³
Ionization energy (second)	11.5 eV		

^aThe inner and outer diameters of the C₆₀ molecule are estimated from (683 – 335 pm) and (683 + 335 pm), respectively, where 335 pm is the interlayer spacing in graphite.

^bFrom the compilation of M. S. Dresselhaus, G. Dresselhaus, and P. C. Eklund, *J. Mater Res.* **1993**, *8*, 2054. Original references given in source.

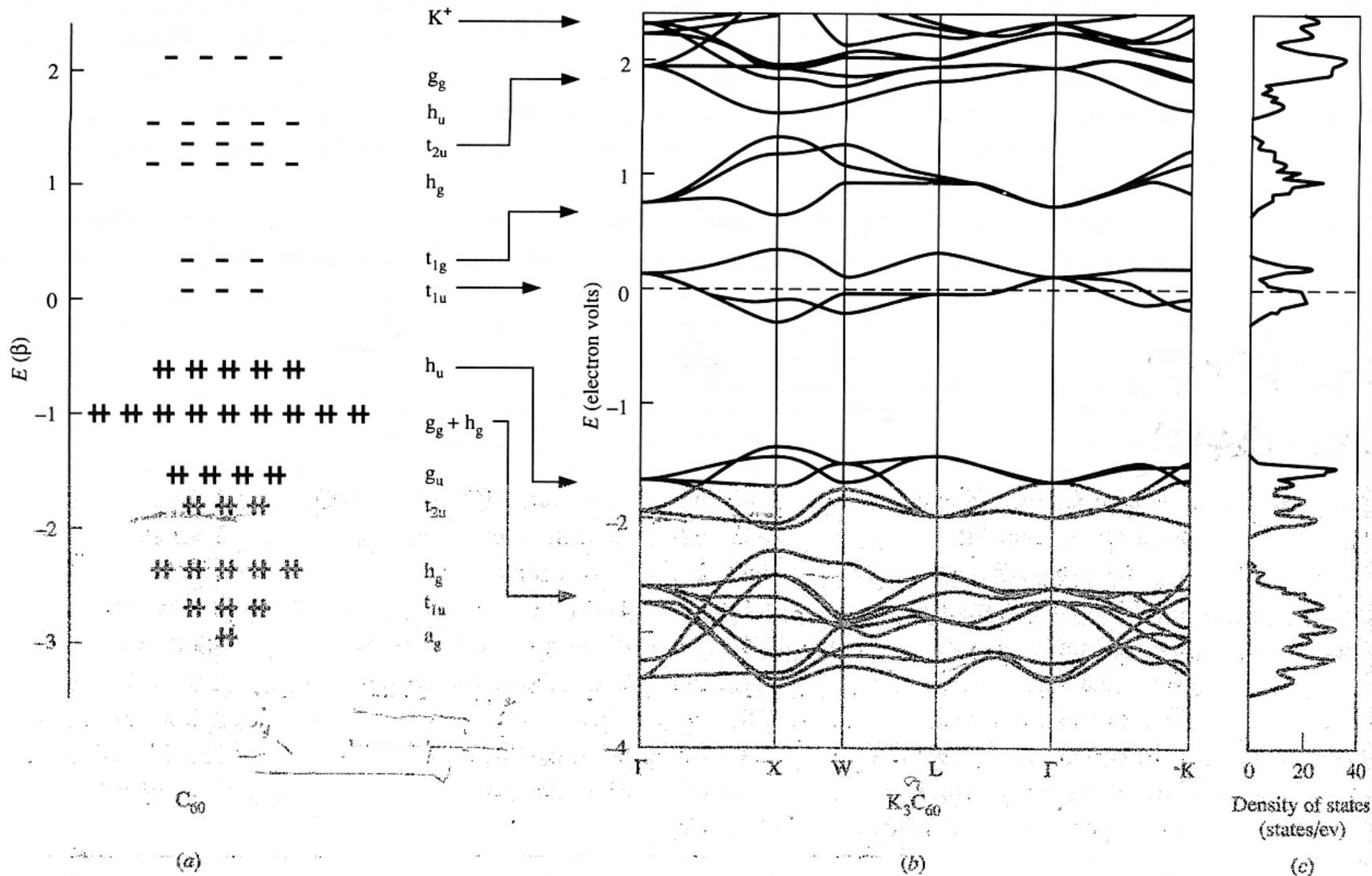


Figure 6.37 (a) Molecular orbitals of C_{60} . (b) Band structure of K_3C_{60} . (c) Corresponding density-of-states curves. (From A. F. Hebard, *Physics Today*, Nov. 1992, 29.)

Electron-doped C60

Na, K, Rb, Cs-doped C60: $T_c \sim 16-18$ K

Hole-doped C60

C60/CHBr₃: $T_c \sim 52$ K

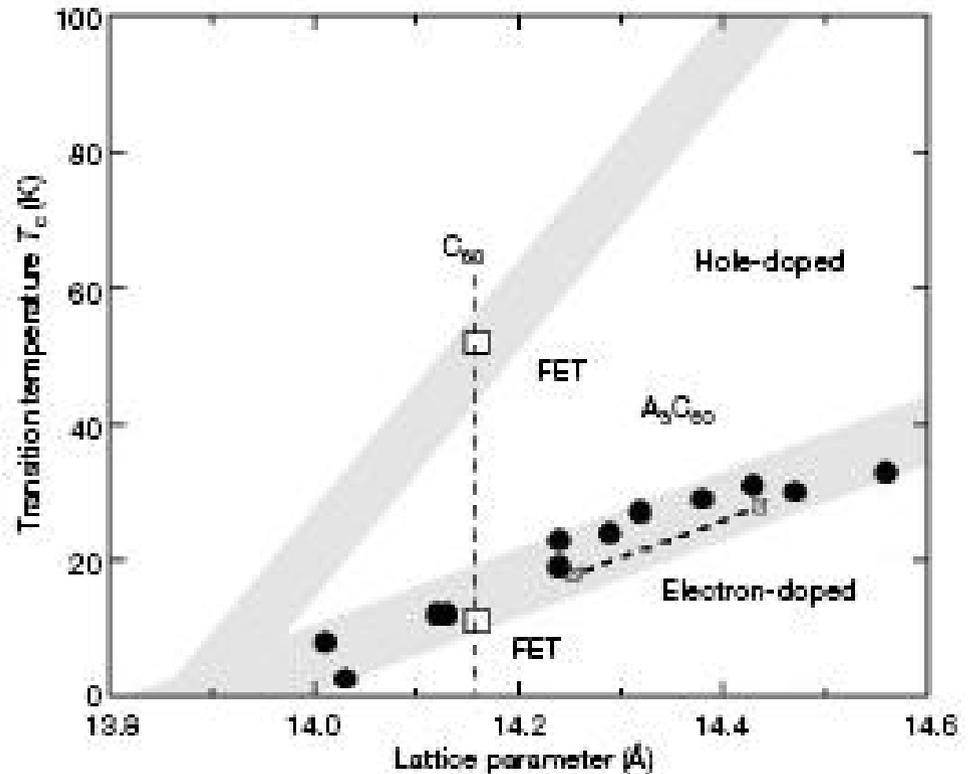


Figure 7. Transition temperature in electron- and hole-doped C60 as function of lattice parameter.

Crystallographic Space Group

230 space group types into 32 crystal classes, one for each associated crystallographic point group. The space groups within each crystal class are characterized by the 14 Bravais lattices which belong to one of 7 crystal systems.

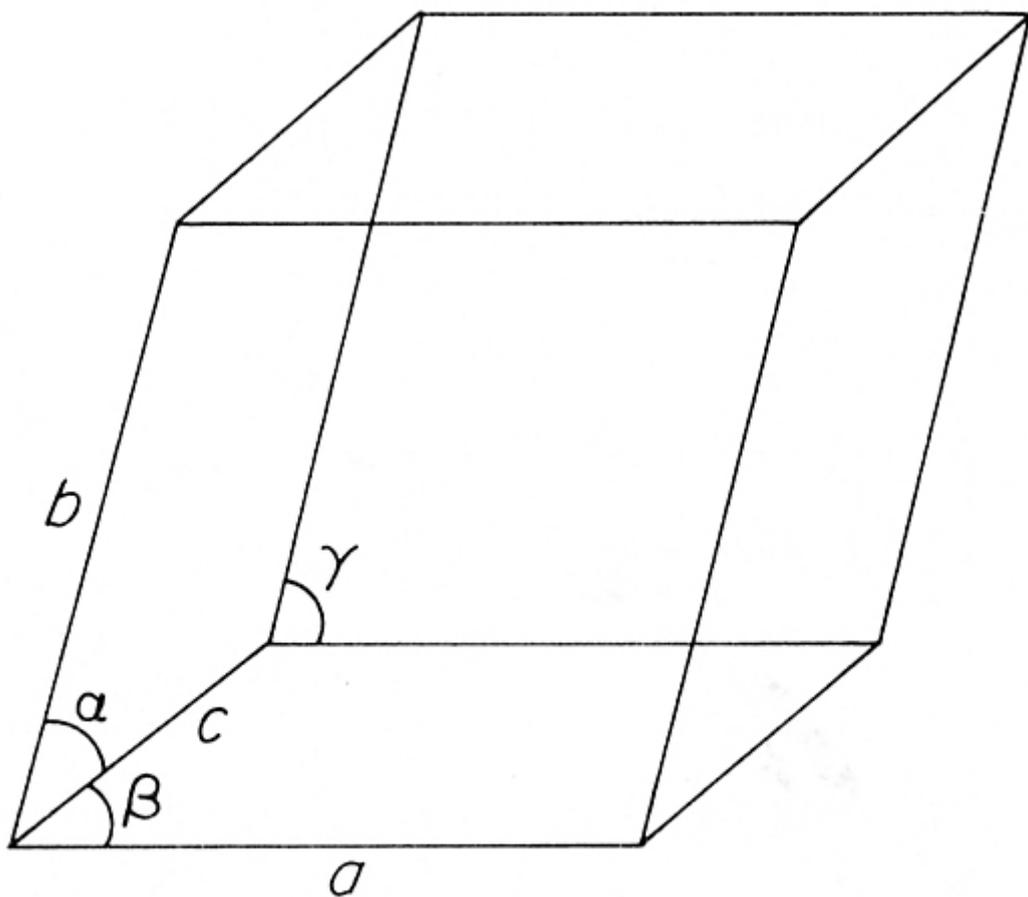


Fig. 1.5a. *Unit cell parameters*

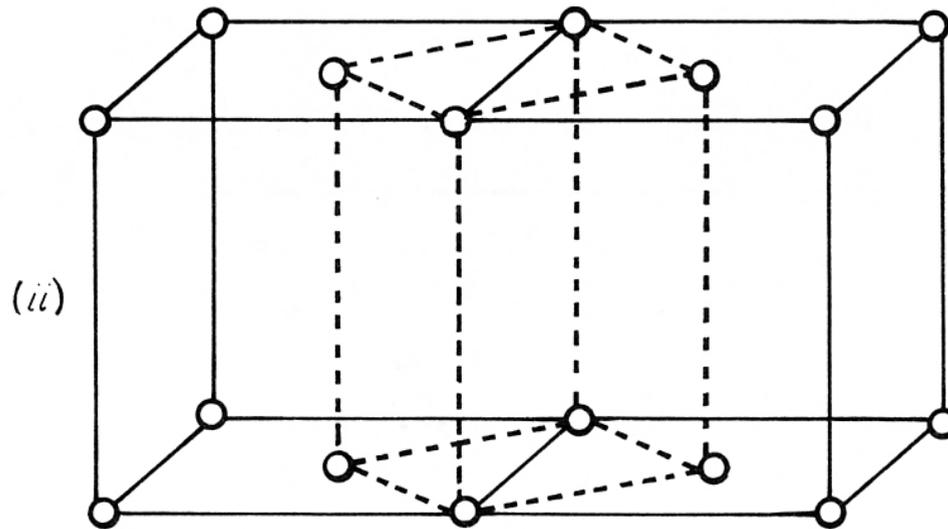
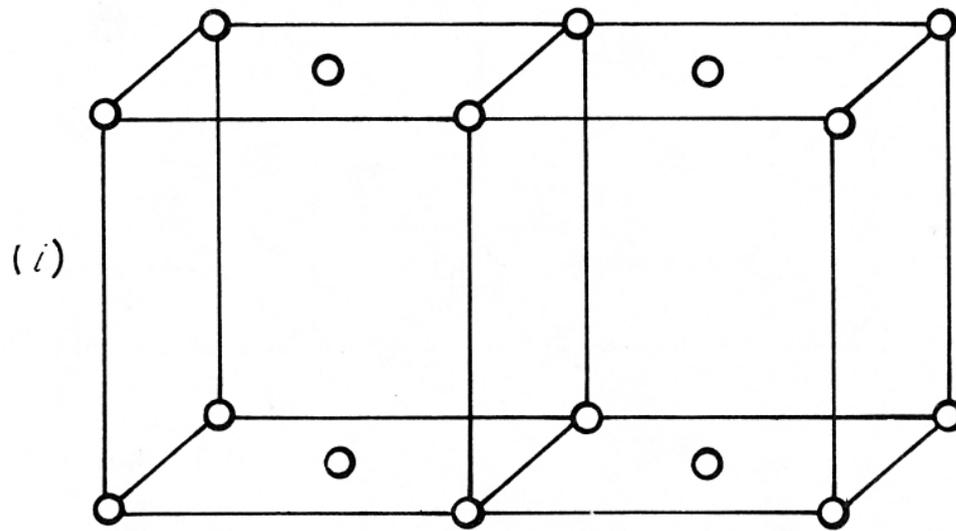
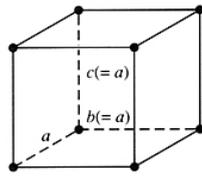
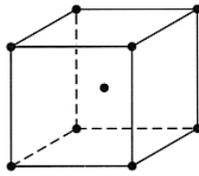


Fig. 1.5d. *Correct choice of unit cell*

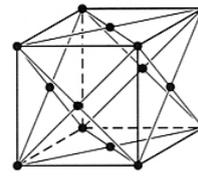
Unit cells of bravais lattices



$a = b = c$
 $\alpha = \beta = \gamma = 90^\circ$
 Primitive (P)

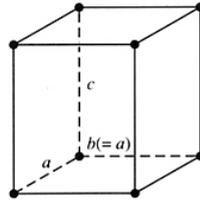


Body centered (I)

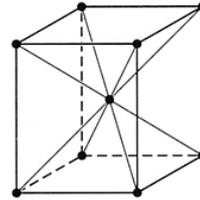


Face centered (F)

Cubic O_h



$a = b \neq c$
 $\alpha = \beta = \gamma = 90^\circ$
 Primitive (P)

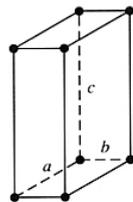


Body centered (I)

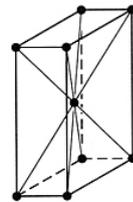
Tetragonal D_{4h}

14 Bravais Lattice

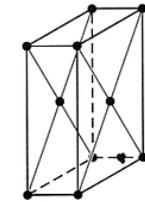
7 Crystal Systems



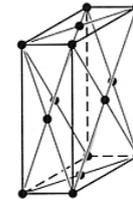
$a \neq b \neq c$
 $\alpha = \beta = \gamma = 90^\circ$
 Primitive (P)



Body centered (I)

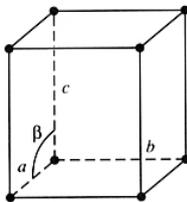


End centered ($C, A, \text{ or } B$)



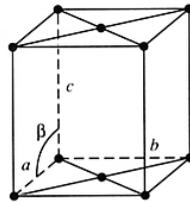
Face centered (F)

Orthorhombic D_{2h}

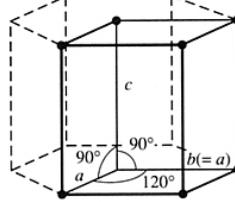


$a \neq b \neq c$
 $\alpha = \gamma = 90^\circ \neq \beta$
 Primitive (P)

Monoclinic

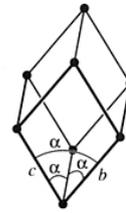


End centered (C)



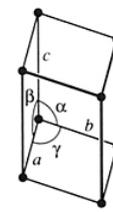
$a = b \neq c$
 $\alpha = \beta = 90^\circ \neq \gamma = 120^\circ$
 Primitive (P)

Hexagonal



$a = b = c$
 $\alpha = \beta = \gamma \neq 90^\circ$
 Primitive (P)

Trigonal or rhombohedral



$a \neq b \neq c$
 $\alpha \neq \beta \neq \gamma \neq 90^\circ$
 Primitive (P)

Triclinic

Table 6.1 Crystal systems

Name	Point group of lattice	Unit cell	Subgroups	Order
Triclinic	$C_1(S_2)$ $\bar{1}$	$a \neq b \neq c$ $\alpha \neq \beta \neq \gamma \neq 90^\circ$	C_1 1 C_i $\bar{1}$	1 2
Monoclinic	C_{2h} $2/m$	$a \neq b \neq c$ $\alpha = \gamma = 90^\circ \neq \beta$	C_{1h} m C_2 2 C_{2h} $2/m$	2 2 4
Orthorhombic	D_{2h} mmm	$a \neq b \neq c$ $\alpha = \beta = \gamma = 90^\circ$	C_{2v} $mm2$ D_2 222 D_{2h} mmm	4 4 8
Tetragonal	D_{4h} $4/mmm$	$a = b \neq c$ $\alpha = \beta = \gamma = 90^\circ$	C_4 4 S_4 $\frac{4}{2}$ C_{4h} $\frac{4}{m}$ D_{2d} $\frac{4}{2}m$ C_{4v} $4mmm$ D_4 422 D_{4h} $4/mmm$	4 4 8 8 8 8 16
Trigonal (rhombohedral)	D_{3d} $\bar{3}m$	$a = b = c$ $120^\circ > \alpha = \beta = \gamma \neq 90^\circ$	C_3 $\frac{3}{2}$ S_6 $\frac{3}{2}$ C_{3v} $3m$ D_3 32 D_{3d} $\bar{3}m$	3 6 6 6 12
Hexagonal	D_{6h} $6/mmm$	$a = b \neq c$ $\alpha = \beta = 90^\circ$ $\gamma = 120^\circ$	C_{3h} $\bar{6}$ C_6 6 C_{6h} $\frac{6}{m}$ D_{3h} $\bar{6}m2$ C_{6v} $6mm$ D_6 622 D_{6h} $6/mmm$	6 6 12 12 12 12 24
Cubic	O_h $m\bar{3}m$	$a = b = c$ $\alpha = \beta = \gamma = 90^\circ$	T 23 T_h $m\bar{3}$ T_d $\bar{4}3m$ O 432 O_h $m\bar{3}m$	12 24 24 24 48

**Miller Indices (hkl): a plane in the
crystal lattice**

$h = a/\text{cutting point along a-axis}$

$k = b/\text{cutting point along b-axis}$

$l = c/\text{cutting point along c-axis}$

Miller Indices (hkl)

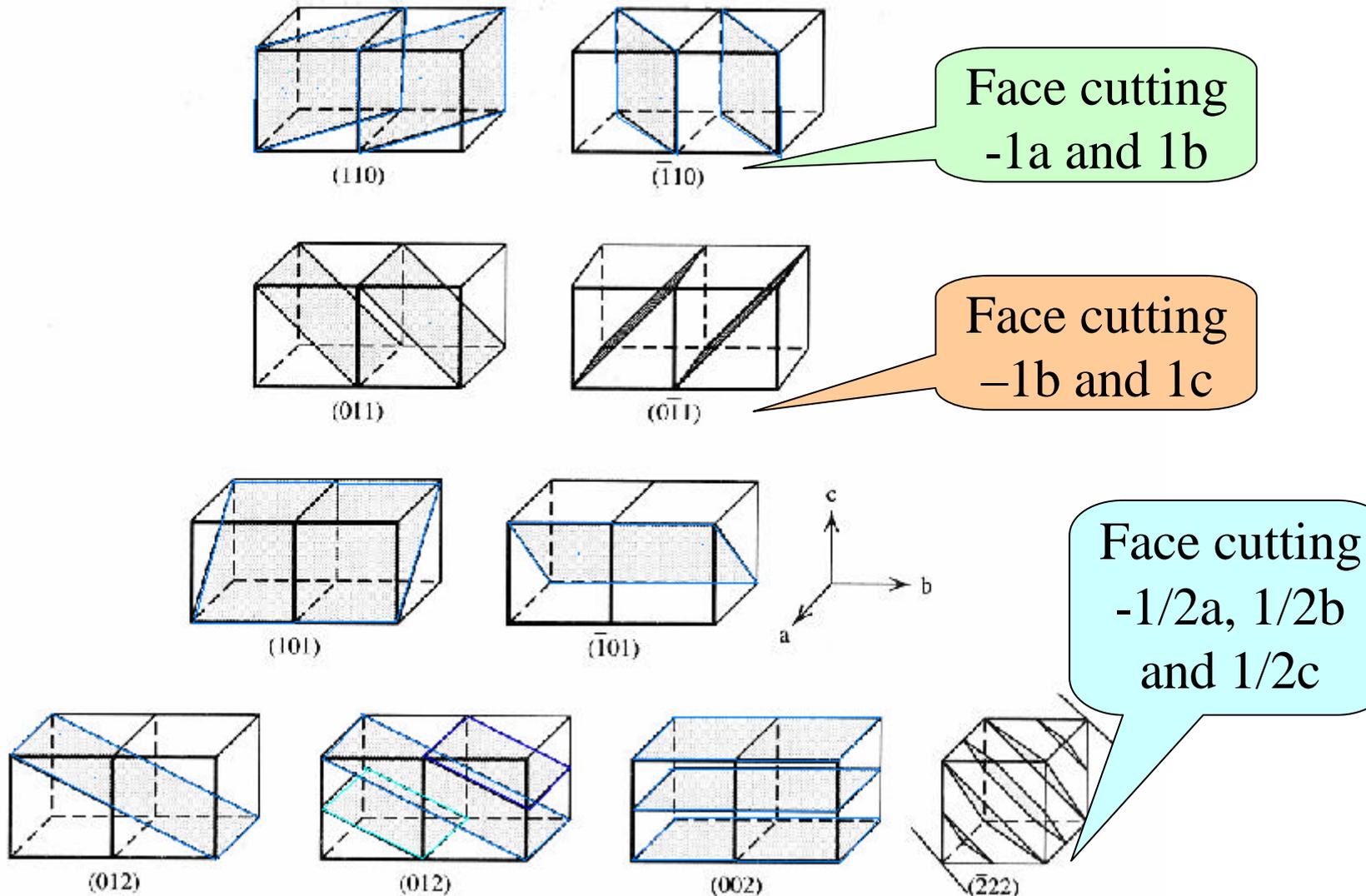


Figure 6.2 Miller indices. The $[110]$ form and others as indicated.

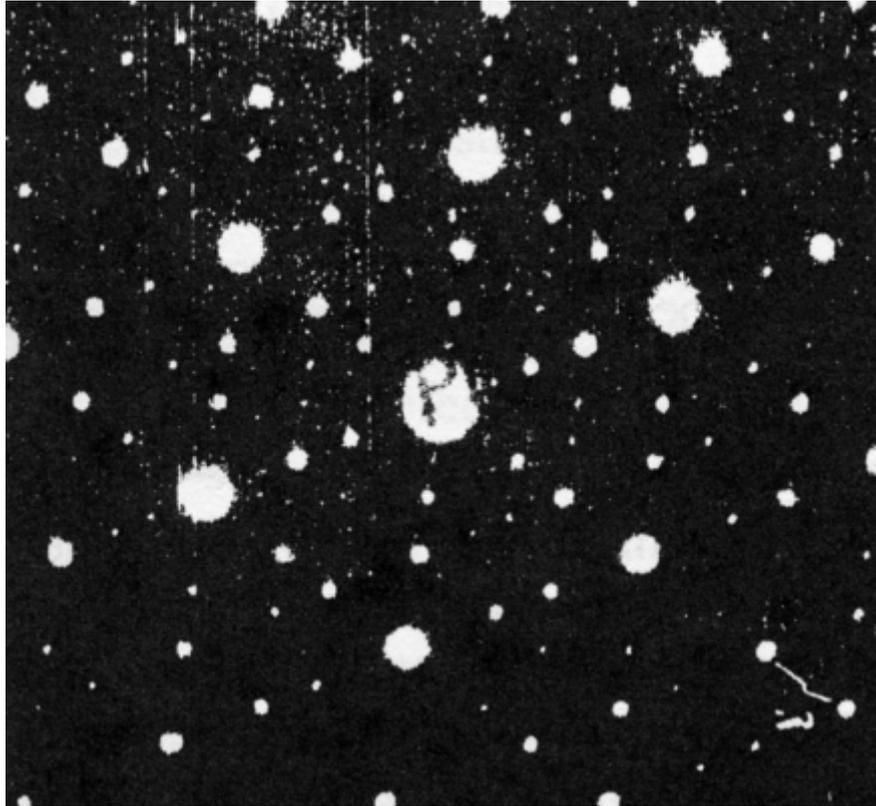
Crystallographic data



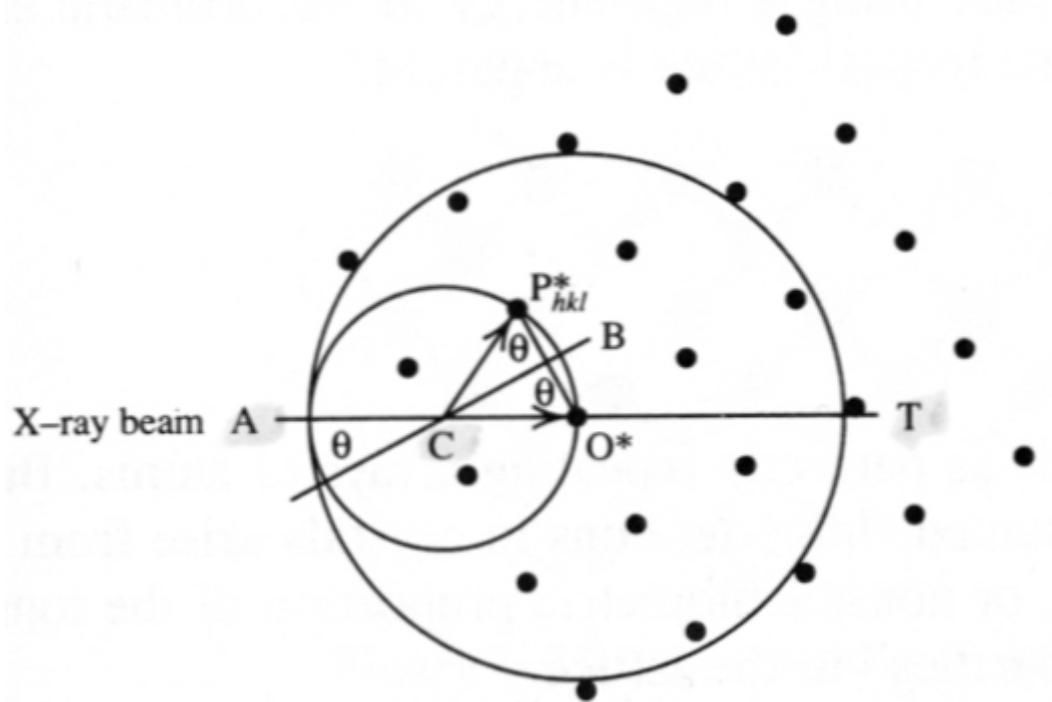
34-268

$\text{CuY}_2\text{Si}_4\text{O}_{12}$	d Å	Int	hkl	d Å	Int	hkl
Copper Yttrium Silicate	7.15	9	020	2.0427	16	241
	6.19	20	110	1.9554	6	260,170
	4.61	14	001	1.9366	4	042
	4.13	14	111	1.8925	4	261
	3.873	40	021	1.8668	1	311
	3.434	4	200	1.8465	14	242
	3.375	55	111	1.8348	4	312
	3.118	100	201	1.8171	5	152
	3.094	20	220	1.7858	9	350,080
	2.823	80	041	1.7764	9	$\bar{3}51$
	2.806	25	131	1.7444	2	$\bar{4}01$
	2.637	25	150	1.7371	10	202
	2.491	14	201	1.7156	10	400
	2.475	30	240	1.6954	6	421
	2.382	3	151,060	1.6877	3	222
	2.362	10	221	1.6685	4	420
	2.323	3	$\bar{1}12$	1.6638	18	152
	2.305	9	002	1.5994	2	262
	2.259	8	310	1.5841	2	280
	2.240	12	311	1.5727	9	351
	2.206	25	151	1.5675	12	$\bar{4}41$
	2.195	5	022	1.5622	12	242
	2.157	10	202	1.5595	14	402
	2.115	3	051	1.5531	5	352
	2.0656	8	222	1.5467	12	440,190
Rad. $\text{CuK}\alpha_1$ λ 1.54056 Filter Mono. d-sp Guinier Cut off Int. Diffractometer $1/1_{\text{cor.}}$ 2.1 Ref. Lambert. Fysel. Mineralogisch-Petrographisches Institut. Uni- versitat Heidelberg Germany. JCPDS Grant-in-Aid Report. (1983)						
Sys. Monoclinic S.G. $C2/c$ a 7.052 b 14.283 c 4.738 A 0.4937 C 0.3317 α β 103.32 γ Z 2 Ref.						
D_x 3.902 D_m 3.980 mp						
Color Light-blue CuO (p.a., Merck) + Y_2O_3 (puriss., Fluka) + 45SiO_2 (Aerosil, De- gussa) heated for three weeks in open Pt-crucible at 1000°C. Space group from powder data. $F_{002} = 81.5(0.009, 42)$ Silicon used as inter- nal standard. PSC: mC38						
See following card						

Diffraction Pattern



<Ref> Structure determination by X-ray crystallography
M.F.C. Ladd and R.A. Palmer, NY : Plenum Press, 1985.



$$2d \sin \theta = n\lambda$$

$$\sin \theta = n\lambda / 2d \quad \Rightarrow \quad \text{Reciprocal lattice}$$

real space (\vec{a} , \vec{b} , \vec{c}) ? **reciprocal space** (\vec{a}^* , \vec{b}^* , \vec{c}^*)

The non-coplanar vectors \vec{a} , \vec{b} , and \vec{c} define a unit cell in the real lattice. The corresponding vectors for the unit cell of a reciprocal lattice \vec{a}^* , \vec{b}^* , and \vec{c}^* will be defined by

$$\vec{a}^* = \frac{1}{\vec{a}} = \frac{\vec{c} \times \vec{b}}{V} \quad \vec{b}^* = \frac{1}{\vec{b}} = \frac{\vec{c} \times \vec{a}}{V} \quad \vec{c}^* = \frac{1}{\vec{c}} = \frac{\vec{a} \times \vec{b}}{V}$$

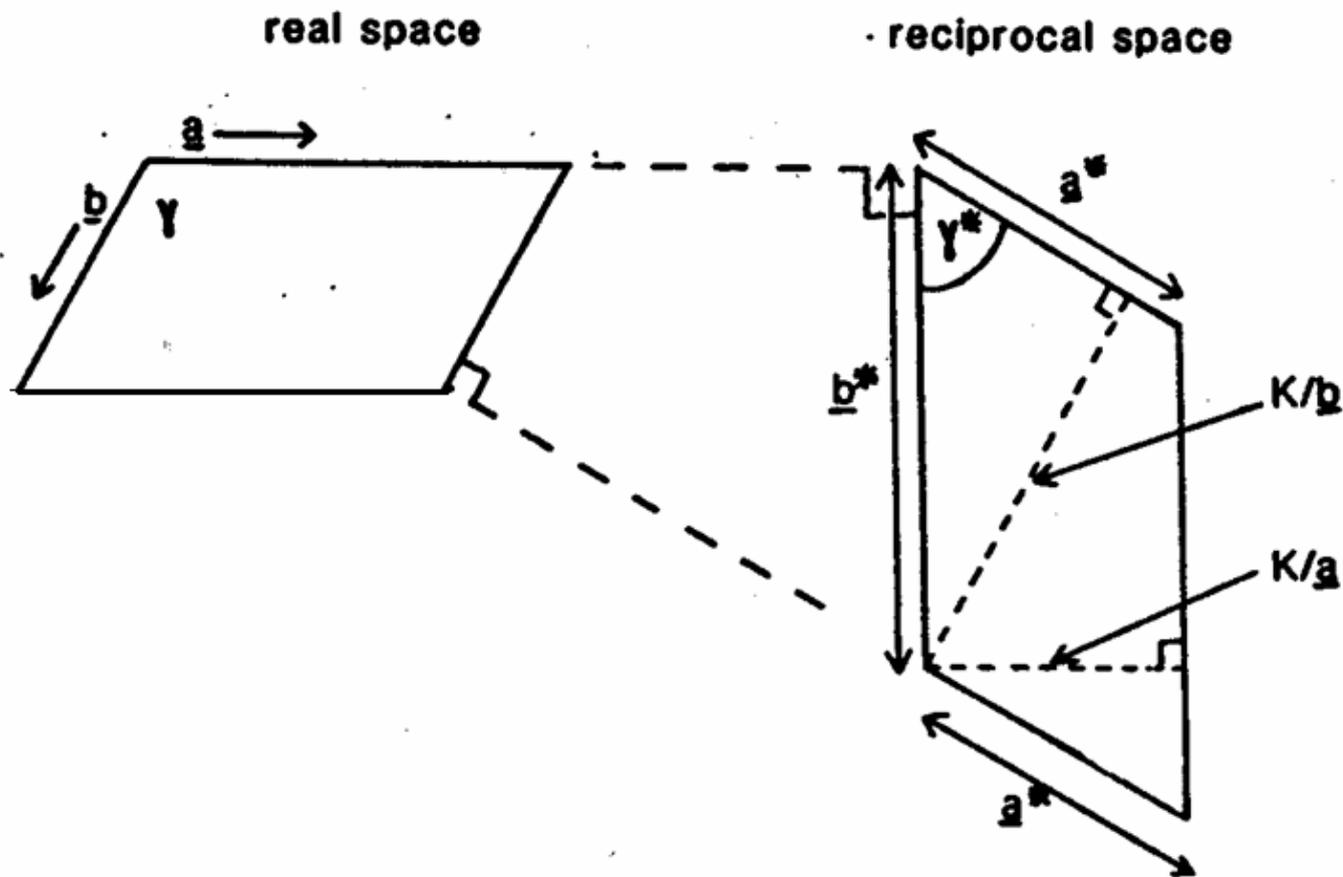
where the unit cell volume V is given by

$$V = \vec{a} \times \vec{b} \times \vec{c} = \frac{1}{V^*}$$

Hence, the reciprocal lattice vectors, \vec{a}^* and \vec{b}^* are normal to the planes **bc**, **ca**, and **ab** respectively in the real space. By the way, the symmetry in the reciprocal space is the same as in the real space.

Relation of the Crystal lattice and Reciprocal lattice

(d)



Reciprocal lattice

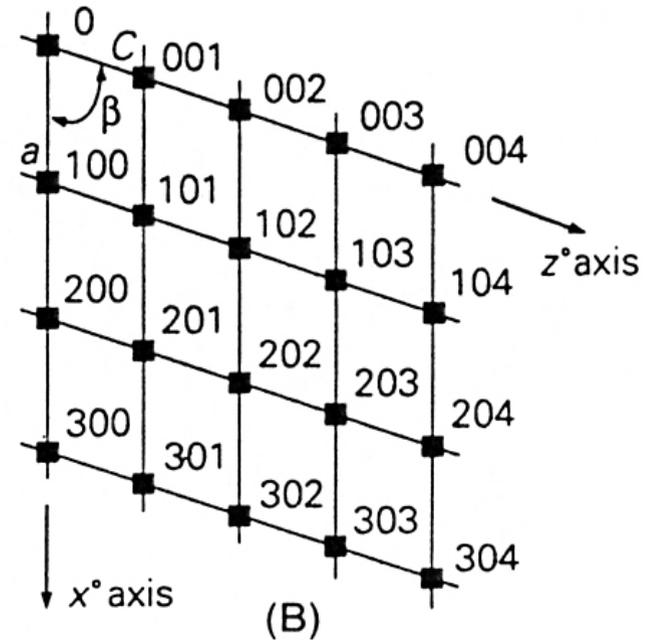
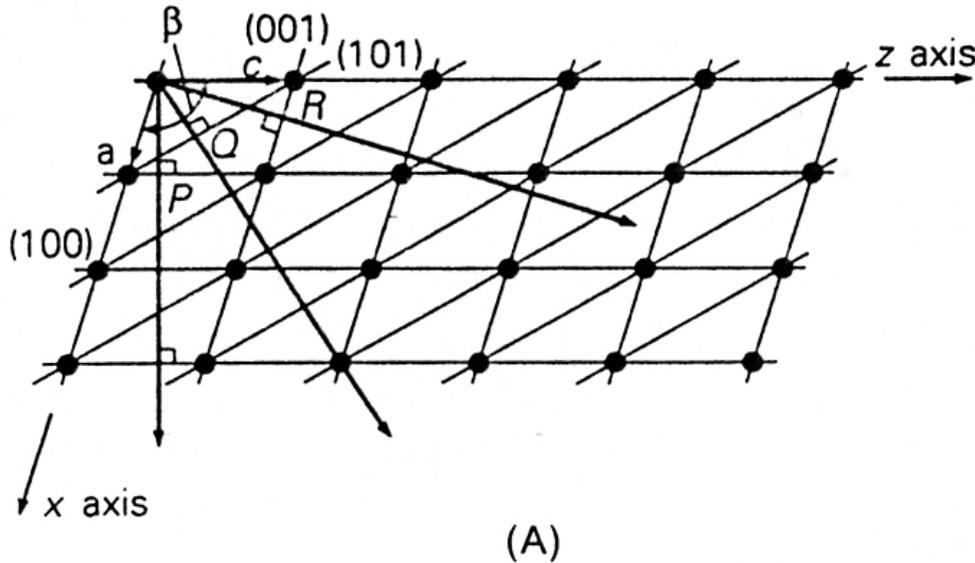


FIGURE 17-2 (A) Direct and (B) reciprocal lattices. (Reproduced with permission from M. F. C. Ladd and R. A. Palmer, "Structure Determination by X-Ray Crystallography." 2nd ed. Plenum Press, New York.)

Stereographic projection

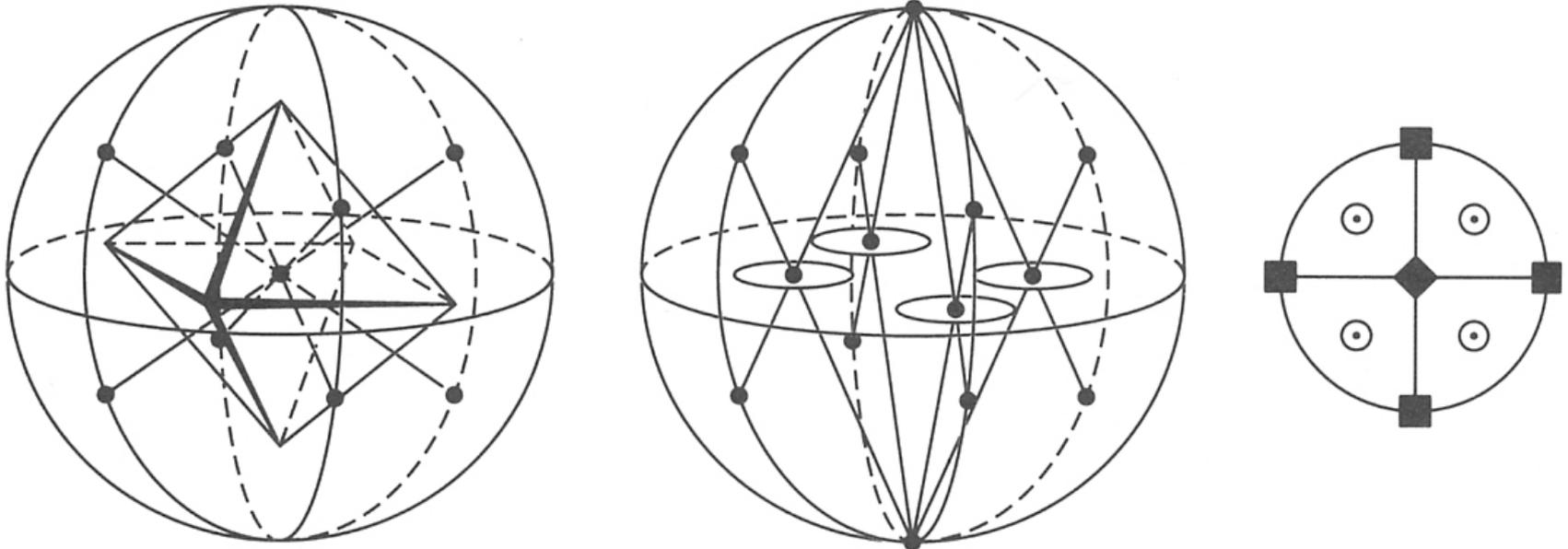


Figure 6.3 Construction of stereographic projection of an octahedron.

Conventional graphic symbols

-  2-fold rotation axis
-  3-fold rotation axis
-  4-fold rotation axis
-  6-fold rotation axis

-  (1-fold) inversion axis
-  2-fold rotation–inversion³ axis
-  3-fold rotation–inversion axis
-  4-fold rotation–inversion axis
-  6-fold rotation–inversion axis

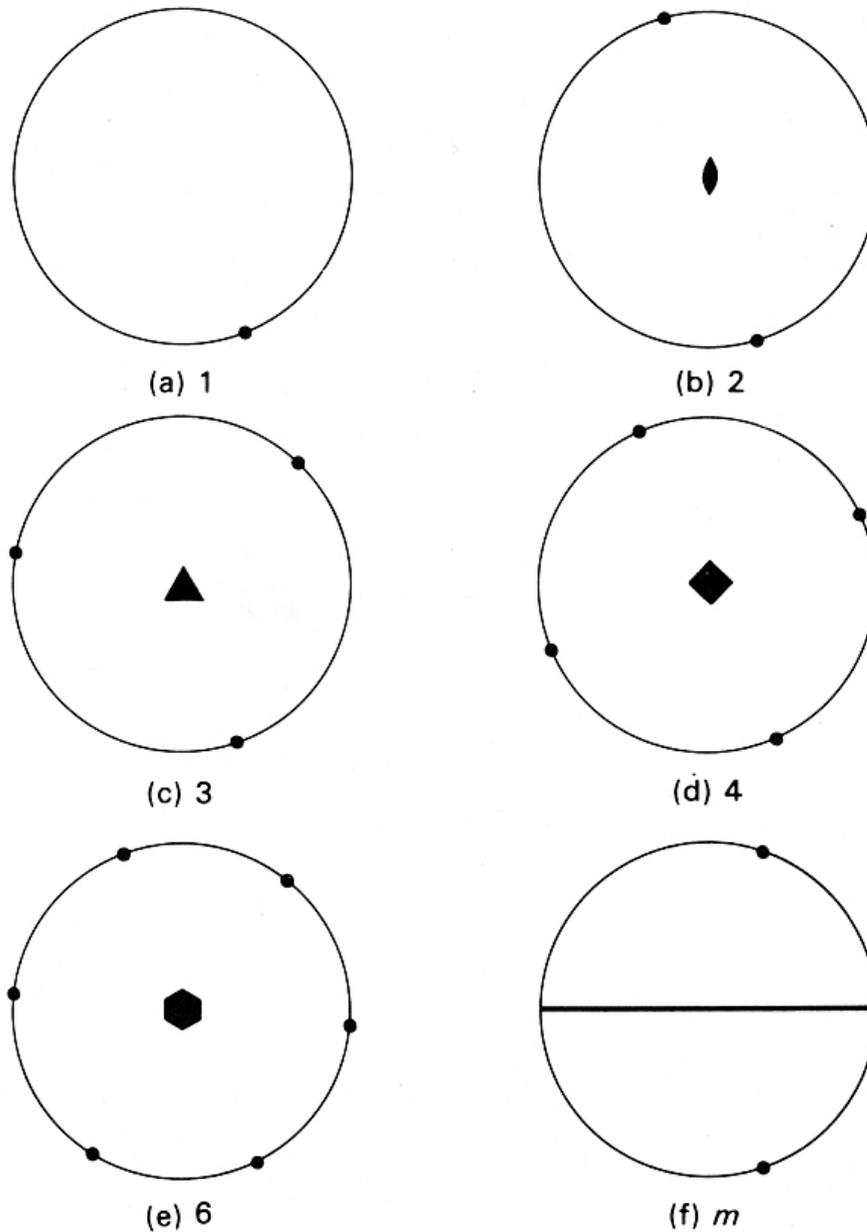


FIGURE 1.28. Stereograms of the point groups of the objects in Figure 1.27; the conventional graphic symbols for R and m are shown.

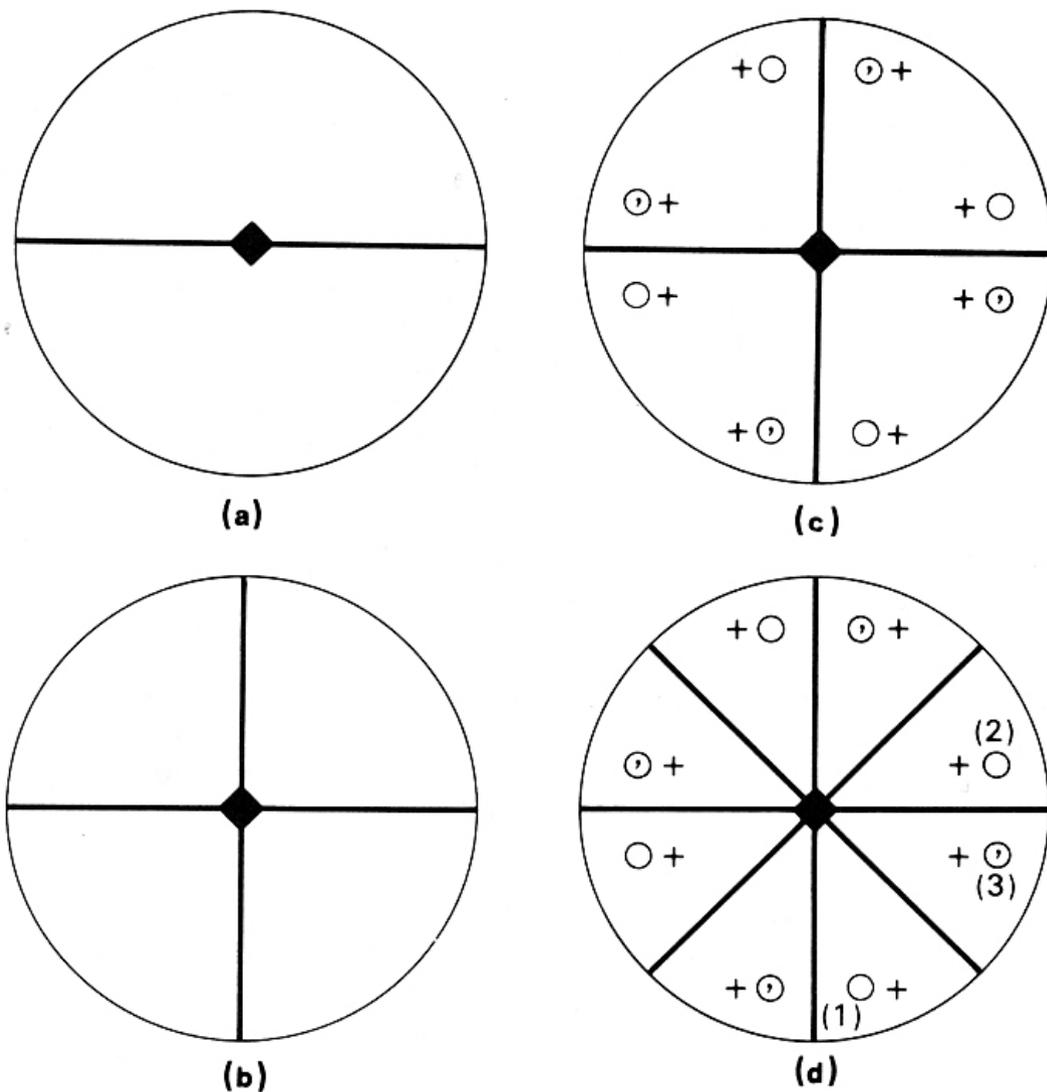


FIGURE 1.36. Intersecting symmetry elements: (a) one m plane is inconsistent with intersecting 4; (b) consistent; (c) points generated by $4m$; (d) complete stereogram ($4mm$).

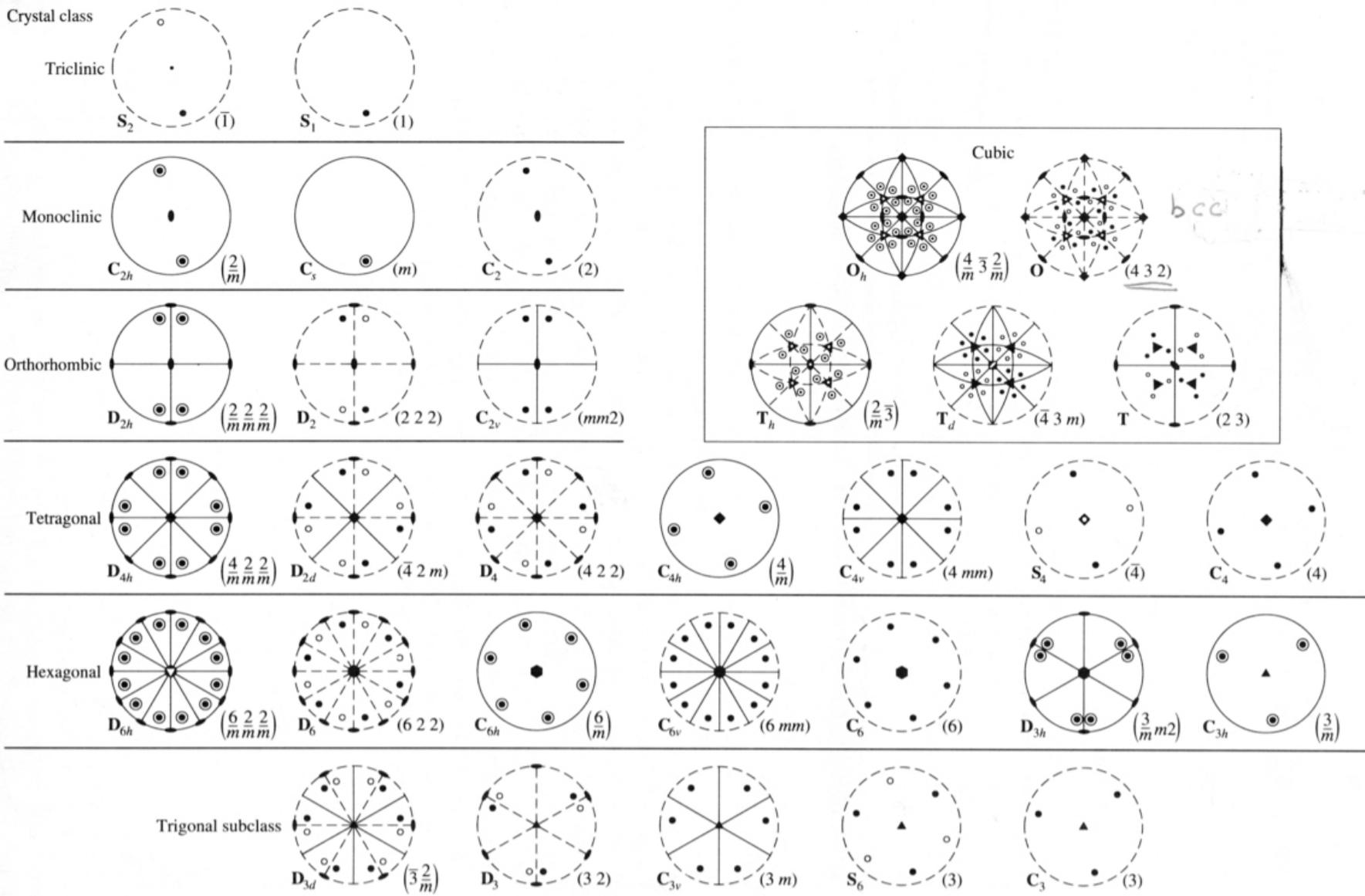
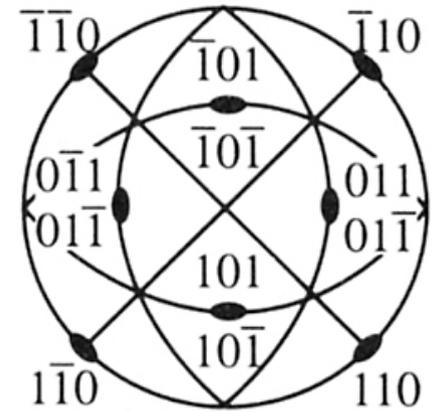
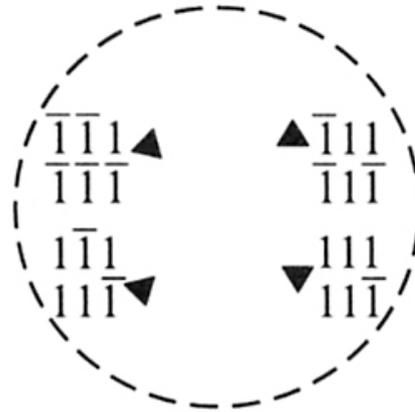
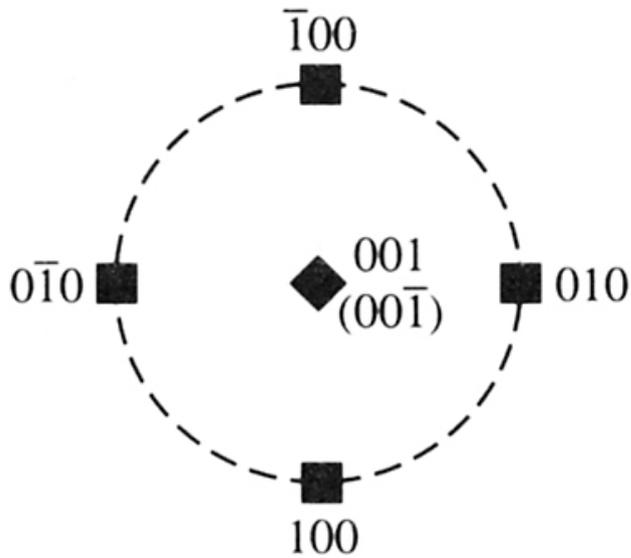
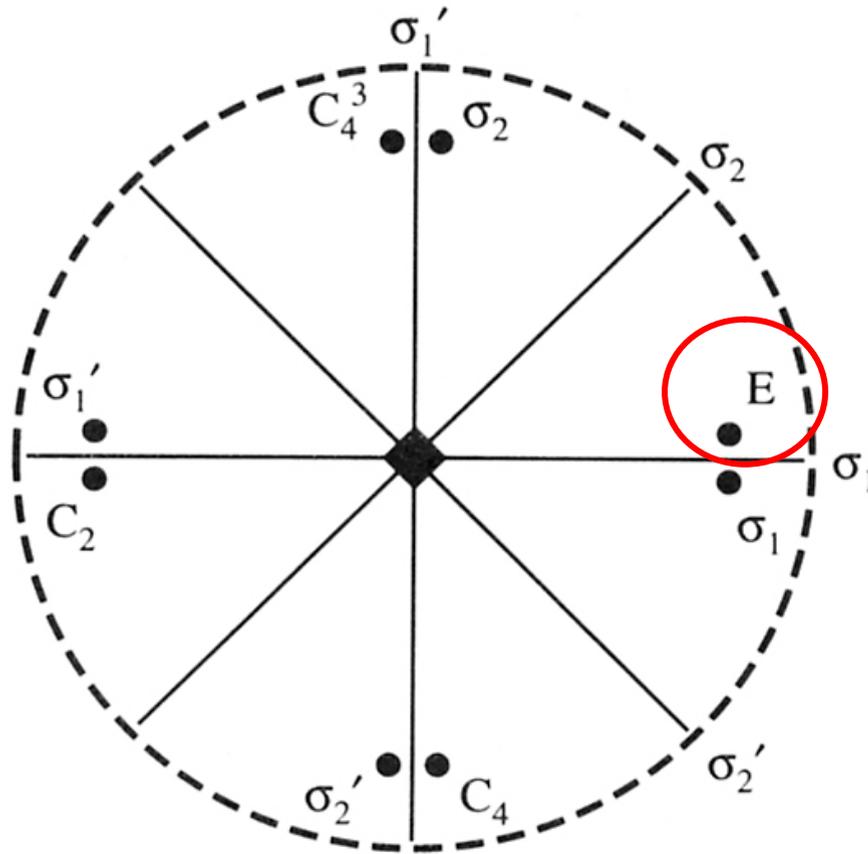


Figure 6.5 Crystallographic point groups.



Special points in the Cubic System (O or 432)

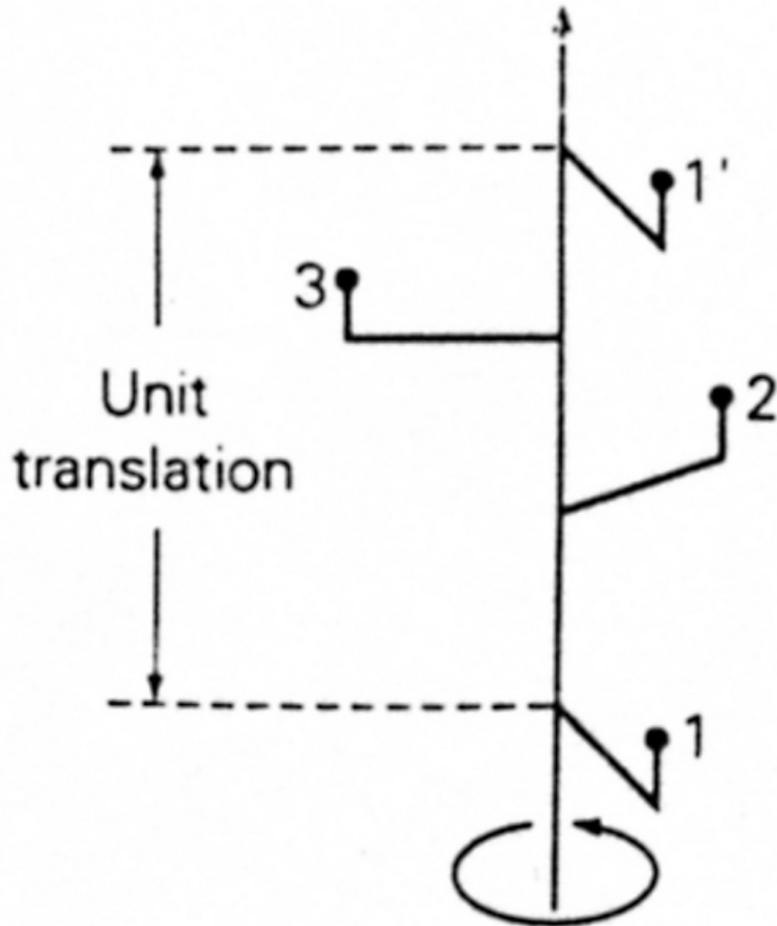
falling on rotational axes along with the Miller indices of the projection plane



Stereogram of the C_{4v} group, with a general point E and the locations to which it is moved by the symmetry operations indicated.

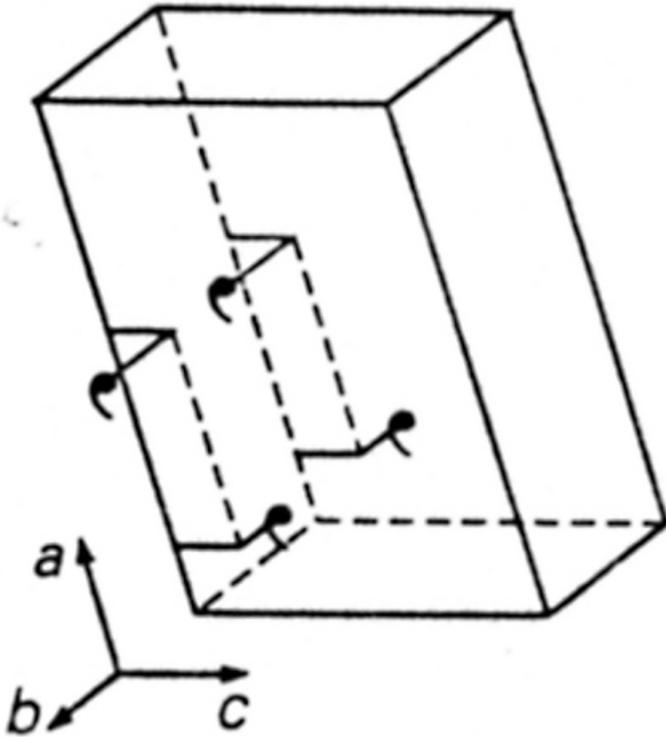
Space Group- containing space symmetry operations

1) Screw Axes



3_1 screw axis-
moving $1/3$ distance
along one unit cell dimension,
then rotating for $2\pi/3$

2) Glide Planes



a glide-
moving $1/2$ distance
along a-axis,
then reflecting versus ac-plane
(or ab-plane)

a, b, c glide-

moving $\frac{1}{2}$ distance along a, b or c-axis,
then reflecting

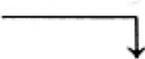
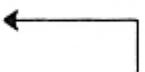
n glide-

moving $\frac{1}{2}$ distance along **face-diagonal**,
then reflecting

d glide-

moving $\frac{1}{4}$ distance along **body-diagonal**,
then reflecting

TABLE 2.5. Glide-Plane Notation

Symbol	Orientation	Graphic symbol		Translational component
		to projection	⊥ to projection	
<i>a</i>	(010) or (001)	-----		<i>a</i> /2
<i>b</i>	(100) or (001)	-----		<i>b</i> /2
<i>c</i>	(100) or (010)	None	<i>c</i> /2
<i>n</i>	(100) (010) (001)	} ----- }	}  }	(<i>b</i> + <i>c</i>)/2 (<i>a</i> + <i>c</i>)/2 (<i>a</i> + <i>b</i>)/2

Lattice Centering—Pure Translation Symmetry

type			position
P	Primitive	1	(0,0,0)
I	Body center	2	(0, 0, 0); (1/2, 1/2, 1/2)
A	Base center	2	(0, 0, 0); (0, 1/2, 1/2)
B		2	(0, 0, 0); (1/2, 0, 1/2)
C		2	(0, 0, 0); (1/2, 1/2, 0)
F	Face center	4	(0, 0, 0); (1/2, 1/2, 0); (1/2, 0, 1/2); (1/2, 1/2, 1)
R	Rhombohedral	3	(0,0,0) ; (2/3, 1/3, 1/3); (1/3, 1/3, 2/3)

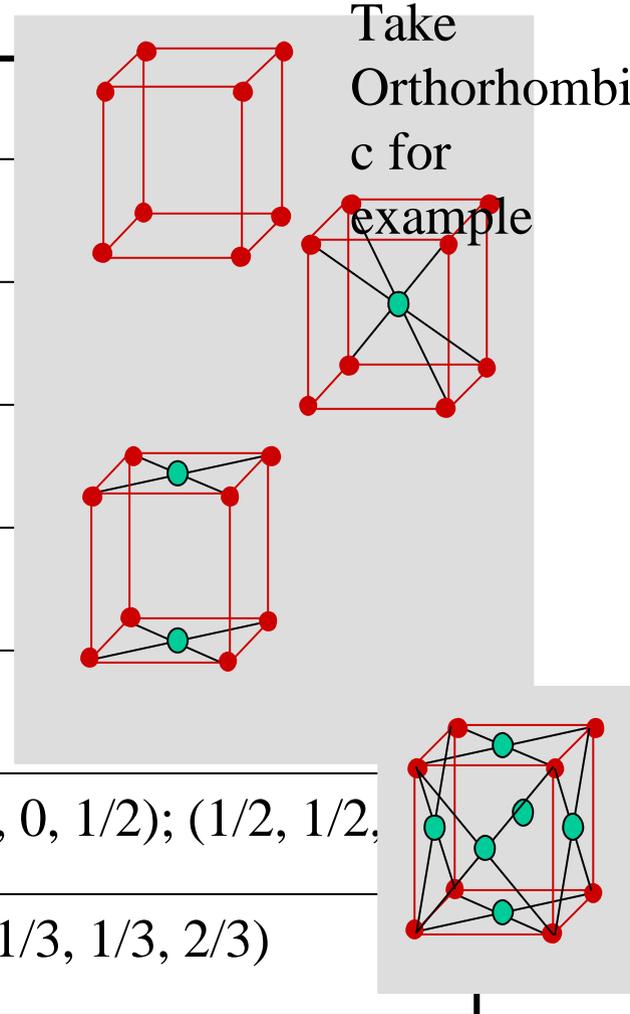


TABLE 17-4. Symmetry Elements and Reflection Conditions

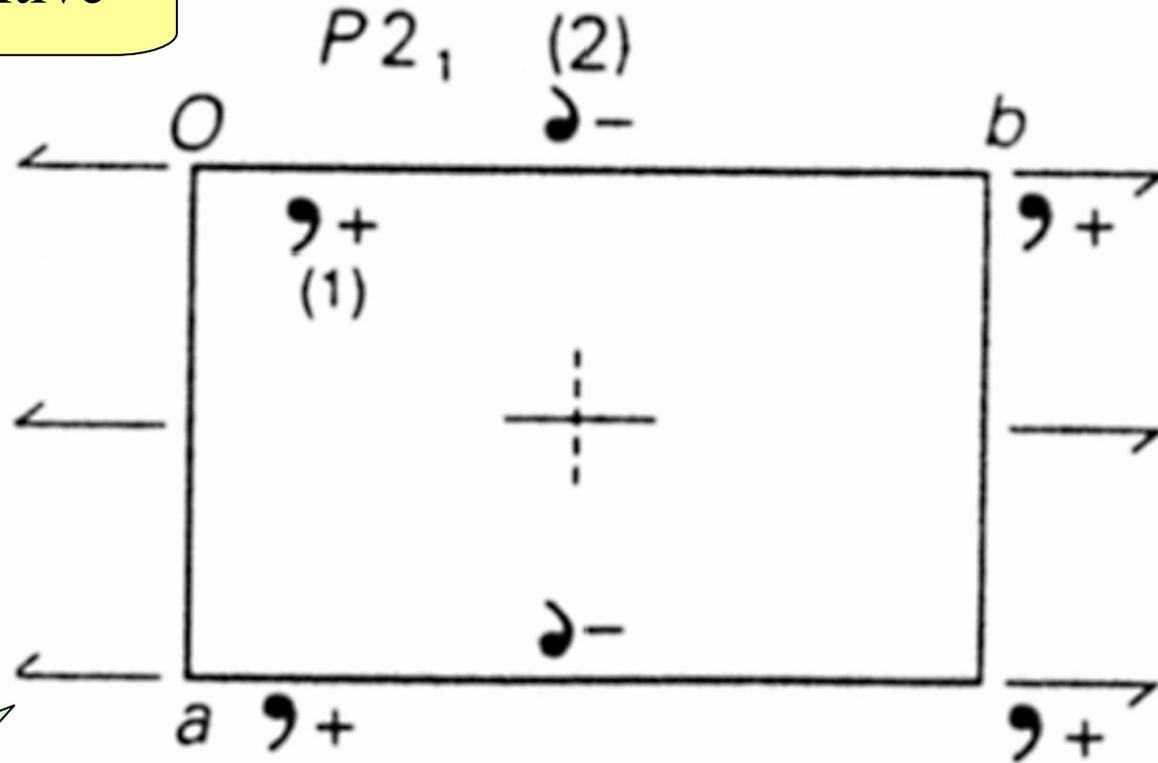
Symmetry Element	Affected Reflection	Condition for Systematic Absence of Reflection
2-fold screw (2_1)	a	$h00$ $h = 2n + 1 = \text{odd}$
4-fold screw (4_2)	b	$0k0$ $k = 2n + 1$
6-fold screw (6_3)	c	$00l$ $l = 2n + 1$
3-fold screw ($3_1, 3_2$)	c^a	$00l$ $l = 3n + 1, 3n + 2,$ <i>i.e., not evenly divisible by 3</i>
6-fold screw ($6_2, 6_4$)		
4-fold screw ($4_1, 4_3$) along	a	$h00$ $h = 4n + 1, 2, \text{ or } 3$
	b	$0k0$ $k = 4n + 1, 2, \text{ or } 3$
	c	$00l$ $l = 4n + 1, 2, \text{ or } 3$
6-fold screw ($6_1, 6_5$) along	c^a	$00l$ $l = 6n + 1, 2, 3, 4, \text{ or } 5$
Glide plane perpendicular to translation $b/2$ (b glide)	a	$0kl$ $k = 2n + 1$
$c/2$ (c glide)		$l = 2n + 1$
$b/2 + c/2$ (n glide)		$k + l = 2n + 1$
$b/4 + c/4$ (d glide)		$k + l = 4n + 1, 2, \text{ or } 3$
Glide plane perpendicular to translation $a/2$ (a glide)	b	$h0l$ $h = 2n + 1$
$c/2$ (c glide)		$l = 2n + 1$
$a/2 + c/2$ (n glide)		$h + l = 2n + 1$
$a/4 + c/4$ (d glide)		$h + l = 4n + 1, 2, \text{ or } 3$
Glide plane perpendicular to translation $a/2$ (a glide)	c	$hk0$ $h = 2n + 1$
$b/2$ (b glide)		$k = 2n + 1$
$a/2 + b/2$ (n glide)		$h + k = 2n + 1$
$a/4 + b/4$ (d glide)		$h + k = 4n + 1, 2, \text{ or } 3$
A -centered lattice (A)	hkl	$k + l = 2n + 1$
B -centered lattice (B)		$h + l = 2n + 1$
C -centered lattice (C)		$h + k = 2n + 1$
Face-centered lattice (F)		$h + k = 2n + 1$
		$h + l = 2n + 1$
		$k + l = 2n + 1$
		$h + k + l = 2n + 1$
		<i>i.e., h, k, l not all even or all odd</i>
Body-centered lattice (I)		$h + k + l = 2n + 1$

^a Note that in the crystal classes in which 3- and 6-fold screws occur as cell axes, these are conventionally assigned to be c , so only the $00l$ reflections need be considered.

Space Group $P2_1$

primitive

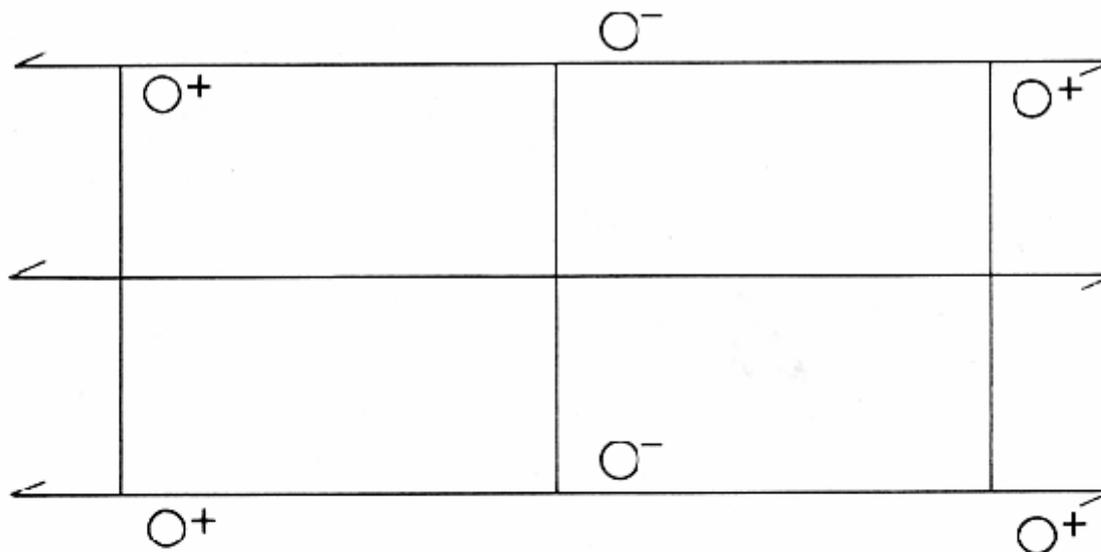
Screw axis 2_1



Screw axis

$P2_1$, equivalent positions

$$(1) x, y, z; (2) \bar{x}, y + \frac{1}{2}, \bar{z}$$



Origin on 2_1

				Limiting conditions	
2	a	1	$x, y, z;$	$\bar{x}, \frac{1}{2} + y, \bar{z}.$	hkl : None
					$h0l$: None
					$0k0$: $k = 2n$

Symmetry of special projections

(001) $pg1(p1g1)$

(100) $p1g(p11g)$

(010) $p2(p211)$

FIGURE 2.30. Space group $P2_1$.

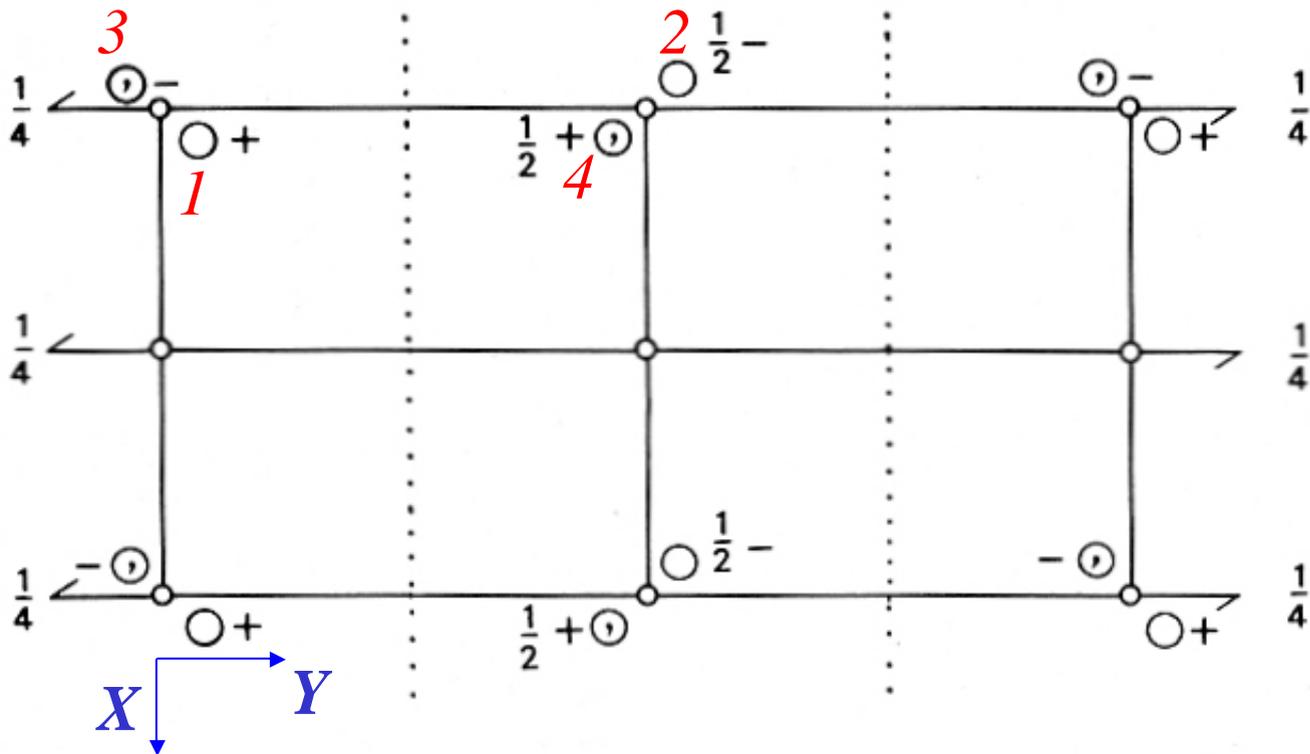
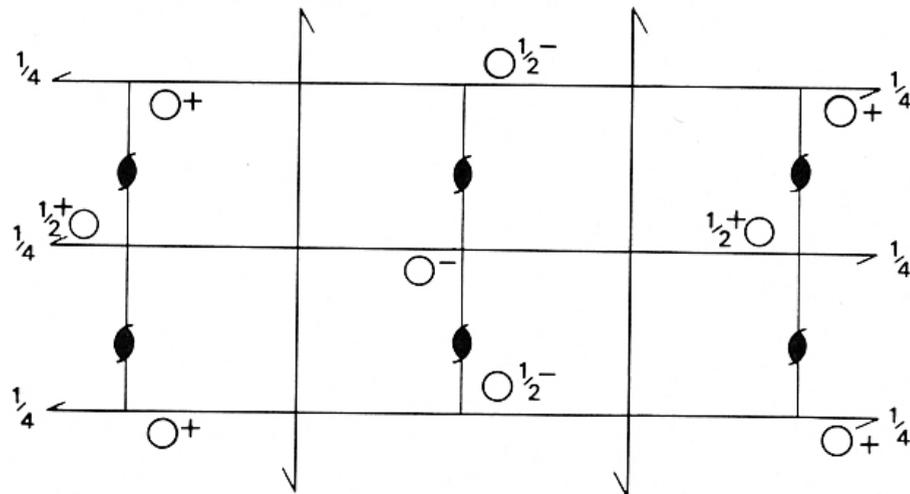


FIGURE 2.32. Space group $P2_1/c$ with the origin on $\bar{1}$ (standard setting).

$\bar{1}$ at $0, 0, 0$ (the origin)

2_1 parallel to the Y -axis, $1/4$ distance from ab -plane c (010), normal to the Y -axis

x, y, z (1) $\textcircled{R} -x, 1/2+y, 1/2-z$ (2) $\textcircled{C} -x, -y, -z$ (3)



Origin halfway between three pairs of non-intersecting screw axes

$$4 \quad a \quad 1 \quad x, y, z; \quad \frac{1}{2}-x, \bar{y}, \frac{1}{2}+z; \quad \frac{1}{2}+x, \frac{1}{2}-y, \bar{z}; \quad \bar{x}, \frac{1}{2}+y, \frac{1}{2}-z.$$

Limiting conditions

$$\left. \begin{array}{l} hkl: \\ 0kl: \\ h0l: \\ hk0: \end{array} \right\} \text{None}$$

$$h00: h = 2n$$

$$0k0: k = 2n$$

$$00l: l = 2n$$

Symmetry of special projections

$$(001) p2gg$$

$$(100) p2gg$$

$$(010) p2gg$$

FIGURE 2.35. Space group $P2_12_12_1$: in space-group diagrams, \bullet represents a 2_1 axis normal to the plane of projection.